Target Tracking and Data Fusion for Ground Situational Awareness

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1.0 INTRODUCTION TO SENSOR DATA FUSION

Sensor data fusion is an omnipresent phenomenon that existed prior to its technological realization or the scientific reflection on it. In fact, all living creatures, including human beings, by nature or intuitively perform sensor data fusion. Each in their own way, they combine or “fuse” sensations provided by different and mutually complementary sense organs with knowledge learned from previous experiences and communications from other creatures. As a result, they produce a “mental picture” of their individual environment, the basis of behaving appropriately in their struggle to avoid harm or successfully reach a particular goal in a given situation.

1.1 Subject Matter

As a sophisticated technology with significant economic and defence implications as well as a branch of engineering science and applied informatics, modern sensor data fusion aims at automating this capability of combining complementary pieces of information. Sensor data fusion thus produces a “situation picture”, a reconstruction of an underlying “real situation”, which is made possible by efficiently implemented mathematical algorithms exploiting even imperfect data and enhanced by new information sources. Emphasis is not only placed on advanced sensor systems, technical equivalents of sense organs, but also on spatially distributed networks of homogeneous or heterogeneous sensors on stationary or moving platforms and on the integration of data bases storing large amounts of quantitative context knowledge. The suite of information sources to be fused is completed by the interaction with human beings, which makes their own observations and particular expertise accessible.

The information to be fused may comprise a large variety of attributes, characterized, for example, by sensor ranges from less than a meter to hundreds of kilometers, by time scales ranging from less than second to a few days, by nearly stationary or rapidly changing scenarios, by actors behaving cooperatively, in-cooperatively, or even hostile, by high precision measurements or sensor data of poor quality.

Sensor data fusion systems emerging from this branch of technology have in effect the character of "cognitive tools", which enhance the perceptive faculties of human beings in the same way conventional tools enhance their physical strength. In this type of interactive assistance system, the strengths of automated data processing (dealing with mass data, fast calculation, large memory, precision, reliability, robustness etc.) are put into service for the human beings involved. Automated sensor data fusion actually enables them to bring their characteristically "human" strengths into play, such as qualitatively correct over-all judgment, expert knowledge and experience, intuition and creativity, i.e. their "natural intelligence" that cannot be substituted by automated systems in the foreseeable future. The user requirements to be fulfilled in a particular application have a strong impact on the actual fusion system design.

1.1.1 Origins of Modern Development

Sensor data fusion systems have been developed primarily for applications, where a particular need for support systems of this type exists, for example in time-critical situations or in situations with a high decision risk, where human deficiencies must be complemented by automatically or interactively working data fusion techniques. Examples are fusion tools for compensating decreasing attention in routine and mass situations, for focusing attention on anomalous or rare events, or complementing limited memory, reaction, and combination capabilities of human beings. In addition to the advantages of reducing the human workload in routine or mass tasks by exploiting large data streams quickly, precisely, and comprehensively, fusion of mutually complementary information sources typically produces qualitatively new and important knowledge that otherwise would remain unrevealed.

The demands for developing such support systems are particularly pressing in defence and security applications, such as surveillance, reconnaissance, threat evaluation, and even weapon control. The earliest examples of large sensor data fusion projects were designed for air defence against missiles and low-flying bombers and influenced the development of civilian air traffic control systems. The development of modern sensor data fusion technology and the underlying branch of applied science was stimulated by the advent of sufficiently powerful and compact computers and high frequency devices, programmable digital signal processors, and last but not least by the "Stratecic Defence Initiative (SDI)" announced by US President RONALD REAGAN on March 23, 1983.

After a certain level of maturity has been reached, the Joint Directors of Laboratories (JDL), an advisory board to the US Department of Defense, coined the technical term "Sensor Data and Information Fusion" in George

Orwell's very year 1984 and undertook the first attempt of a scientific systematization of the new technology and the research areas related to it [1, Chapter 2, p. 24]. To the present day, the scientific fusion community speaks of the "JDL Model of Information Fusion" and its subsequent generalizations and adaptations [1, Chapter 3], [2]. The JDL model provides a structured and integrated view on the complete functional chain from distributed sensors, data bases, and human reports to the users and their options to act including various feed-back loops at different levels (Figure 1.1). It seems to be valid even in the upcoming large fields of civilian applications of sensor data fusion and computer security [3]. Obviously, the fundamental concepts of sensor data fusion have been developed long before their full technical feasibility and robust realizability in practical applications.

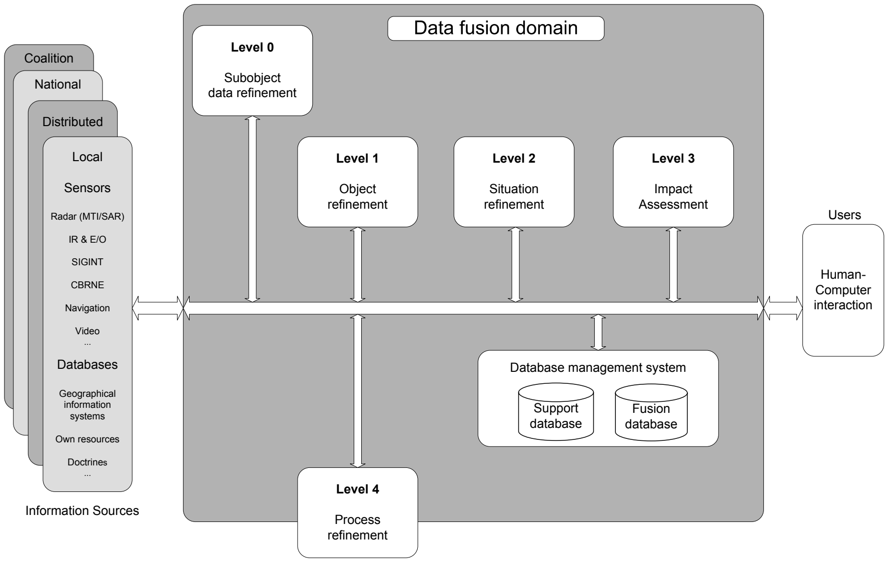

Fig. 1.1. Overview of the JDL-Model of Sensor Data and Information Fusion [1, Chapter 3], which provides a structured and integrated view on the complete functional chain from distributed sensors, data bases, and human reports to the users and their options to act including various feed-back loops at different levels.

1.1.2 General Technological Prerequisites

The modern development of sensor data fusion systems was made possible by substantial progress in the following areas over the recent decades:

1. Advanced and robust *sensor systems*, technical equivalents of sense organs with high sensitivity or coverage are made available that may open dimensions of perception usually unaccessible to most living creatures.
2. *Communication links* with sufficient bandwidths, small latencies, stable connectivity, and robustness against interference are the backbones of spatially distributed networks of homogeneous or heterogeneous sensors.
3. Mature *navigation systems* are prerequisites of (semi-)autonomously operating sensor platforms and common frames of reference for the sensor data based on precise space-time registration including mutual alignment.
4. *Information technology* provides not only sufficient processing power for dealing with large data streams, but also efficient data base technology and fast algorithmic realizations of data exploitation methods.
5. *Technical interoperability*, the ability of two or more sub-systems or components to exchange and to information, is inevitable to build distributed "systems of systems" for sensor exploration and data exploitation [4].
6. Advanced and ergonomically efficient *Human-Machine Interaction (HMI)* tools are an integral part of man-machine-systems presenting the results of sensor data fusion systems to the users in an appropriate way [5].

The technological potential enabled by all these capabilities is much enhanced by integrating then in an overlay sensor data fusion system.

1.1.3 Relation to Information Systems

According to this technological infrastructure, human decision makers on all levels of hierarchy, as well as automated decision making systems, have access to vast amounts of data. In order to optimize use of this high degree of data availability in various decision tasks, however, the data continuously streaming in must not overwhelm the human beings, decision making machines, or actuators involved. On the contrary, the data must be fused in such a way that at the right instant of time the right piece of high-quality information relevant to a given situation is transmitted to the right user or component and appropriately presented. Only if this is the case, can the data streams support goal-oriented decisions and coordinated action planing in practical situations and on all levels of decision hierarchy.

In civilian applications, management information or data warehouse systems are designed in order to handle large information streams. Their equivalents in the defence and security domain are called C^{4}ISTAR Systems [4]. This acronym denotes computer-assisted functions for C^4 (Command, Control, Communications, Computers), I (Intelligence), and STAR (Surveillance, Target Acquisition and Reconnaissance) in order to enable the coordination of defence-related operations. While management information or data warehouse systems are primarily used to obtain competitive advantages in economic environments, C^{4}ISTAR systems aim at information dominance over potential opponents. The observation that more or less the same terminology is used in both areas for characterizing the struggle to avoid harm or successfully reach goals, is an indication of far-reaching fundamental commonalities of decision processes in defence command & control as well as in product development and planing, in spite of different accentuations in particular aspects.

A basic component of C^4ISTAR information systems, modular and flexibly designed as "systems of systems", is the combination of sensor systems and data bases with appropriate sensor data and information fusion sub-systems. The objective at this level is the production of timely, consistent and, above all, sufficiently complete and detailed "situation pictures", which electronically represent a complex and dynamically evolving overall scenario in the air, on the ground, at sea, or in an urban environment. The concrete operational requirements and restrictions in a given application define the particular information sources to be considered and data fusion techniques to be used.

A Characteristic Example

A particularly mature example of an information system, where advanced sensor data fusion technology is among its central pillars, is given by a distributed, coalition-wide C^4ISTAR system of systems for wide-area ground surveillance. It mirrors many of the aspects previously addressed and has been carried out within the framework of a multinational technology program called MAJIIC (Multi-Sensor Aerospace-Ground Joint ISR Interoperability Coalition) [4, Chapter 20]. By collaboratively using interoperable sensor and data exploitation systems in coalition operations, MAJIIC has been designed to improve situational awareness of military commanders over the various levels of the decision making hierarchy.

Based on appropriate concepts of deployment and the corresponding tactical procedures, technological tools for Collection, Coordination and Intelligence Requirements Management (CCIRM) are initiated by individual sensor service requests of deployed action forces. The CCIRM tools produce mission plans according to superordinate priorities, task sensor systems with appropriate data acquisition missions, initiate data exploitation and fusion of the produced sensor data streams in order to obtain high-quality reconnaissance information, and, last but not least, guarantee the feedback of the right information to the requesting forces at the right instant of time.

Under the constraint of leaving existing C^4ISTAR system components of the nations participating in MAJIIC unchanged as far as possible, the following aspects are addressed with particular emphasis:

1. The integration of advanced sensor technology for airborne and ground-based wide-area surveillance is mainly based on Ground Moving Target Indicator Radar (GMTI), Synthetic Aperture Radar (SAR), electro-optical and infrared sensors (E/O, IR) producing freeze and motion imagery, Electronic Support Measures (ESM), and artillery localization sensors (radar- or acoustics-based).

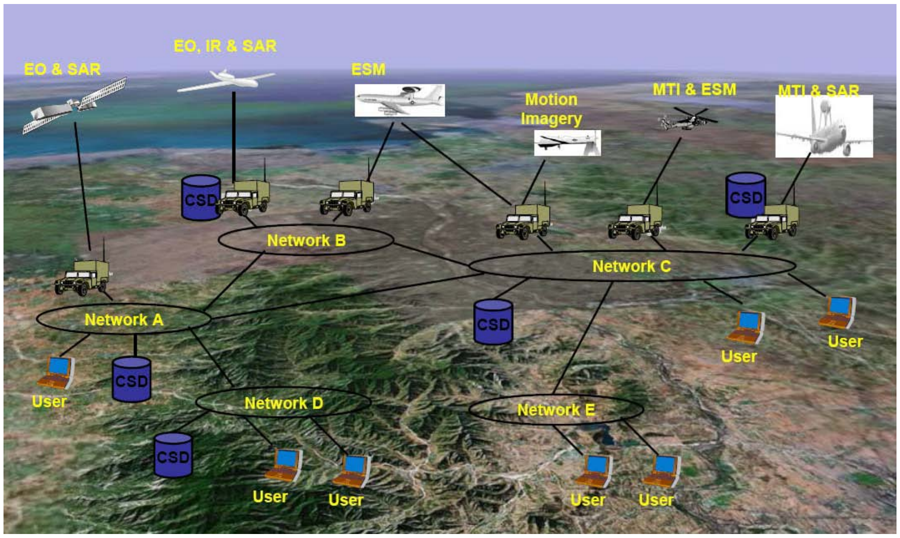

Fig. 1.2. MAJIIC system architecture emphasizing the deployed sensors, databases, and distributed sensor data fusion systems (Interoperable ISR Exploitation Stations).

2. Another basic issue is the identification and implementation of common standards for distributing sensor data from heterogeneous sources including appropriate data and meta-data formats, agreements on system architectures as well as the design and implementation of advanced information security concepts.
3. In addition to sensor data fusion technology itself, tools and procedures have been developed and are continuously enhanced for co-registration of heterogeneous sensors, cross-cueing between the individual sensors of a surveillance system, the sensors of different systems, and between sensors and actuators, as well as for exploitation product management, representation of the “Coalition Ground Picture”, for coordinated mission planning, tasking, management, and monitoring of the MAJIIC sub-systems.
4. MAJIIC-specific communications have been designed to be independent of network-types and communication bandwidths, making it adaptable to varying requirements. Commercially available and standardized internet- and crypto-technology has been used in both the network design and the implementation of interfaces and operational features. Important functionalities are provided by collaboration tools enabling ad-hoc communication between operators and exchange of structured information.
5. The central information distribution nodes of MAJIIC C^4ISTAR system of systems are so-called Coalition Shared Data servers (CSD) making use of modern database technology. Advanced Data Mining and Data Retrieval tools are part of all MAJIIC data exploitation and fusion systems.

6. From an operational point of view, a continuous interaction between Concept Development and Experimentation (CD&E process, [7]) by planning, running, and analyzing simulated and live C^4ISTAR experiments is an essential part of the MAJIIC program, fostering the transfer of MAJIIC capabilities into national and coalition systems.

Figure 1.2 provides an overview of the MAJIIC system architecture and the deployed sensor systems.

1.2 From Imperfect Data to Situation Pictures

Sensor data fusion typically provides answers to questions related to objects of interest such as: Do object exist at all and how many of them are moving in the sensors' fields of view? Where are they geolocated at what time? Where will they be in the future with what probability? How can their overall behavior be characterized? Are anomalies or hints to their possible intentions recognizable? What can be inferred about the classes the objects belong to or even their identities? Are there clues for characteristic interrelations between individual objects? In which regions do they have their origin? What can be said about their possible destinations? Are there observable over-all object flows? Where are sources or sinks of traffic? and many other questions.

The answers to those questions are the constitutive elements, from which near real-time situation pictures can be produced that electronically represent a complex and dynamically evolving overall scenario in the air, on the ground, at sea, under water, as well as in out- or in-door urban environments, and even more abstract spaces. According to the previous discussion, these "situation elements" must be gained from the currently received sensor data streams while taking into account all the available context knowledge and pre-history. Since situation pictures are fundamental to any type of computer-aided decision support, the requirements of a given application define which particular information sources are to be fused.

The sensor data to be fused are usually inaccurate, incomplete, or ambiguous. Closely-spaced moving objects are often totally or partially irresolvable. The measured object parameters may be false or corrupted by hostile measures. The context information is in many cases hard to formalize and even contradictory in certain aspects. These deficiencies of the information to be fused are unavoidable in any real-world application. Therefore, the extraction of 'information elements' for situation pictures is by no means trivial and requires a sophisticated mathematical methodology for dealing with imperfect information. Besides a precise requirement analysis, this is one of the major scientific features that characterizes and shapes sensor data fusion as branch of applied science.

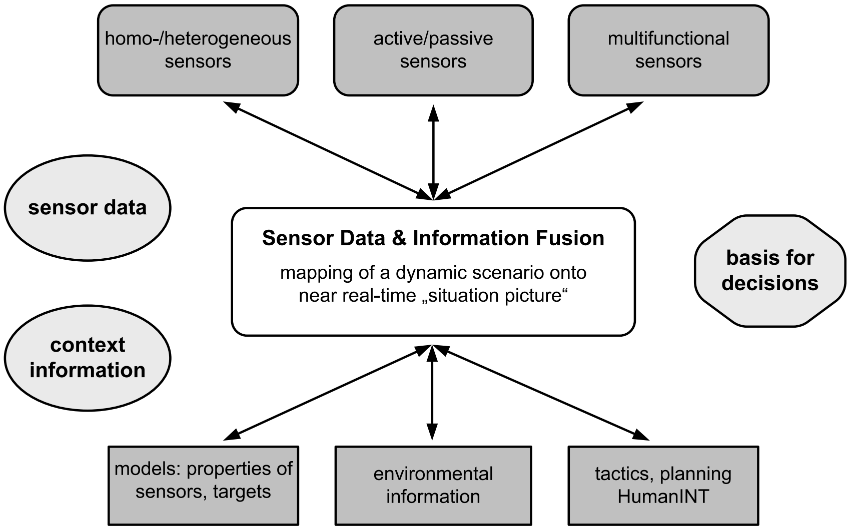

Fig. 1.3. Sensor data and information fusion for situation pictures: overview of characteristic aspects and their mutual interrelation.

1.2.1 Discussion of Characteristic Aspects

Figure 1.3 provides an overview of different aspects within this context and their mutual interrelation, which should be emphasized::

1. The underlying sensor systems can be located in different ways (collocated, distributed, mobile) producing measurements of the same or of different type. A multisensor system potentially increases the coverage or data rate of the total system and may help to resolve ambiguities.
2. Even by fusing homogeneous sensors, information can be obtained that is unaccessible to each sensor individually, such as in stereoscopic vision, where range information is provided by fusing two camera images taken from different viewpoints.
3. Fusion of heterogeneous sensor data is of particular importance, such as the combination of kinematic measurements with measured attributes providing information on the classes to which objects belongs to. Examples for measured attributes are Signal Intelligence (SIGINT), Jet Engine Modulation (JEM), radial or lateral object extension, chemical signatures etc.
4. Especially for defense and security applications, the distinction between active and passive sensing is important as passive sensors enable covert surveillance, which does not reveal itself by actively emitting radiation.

5. Multi-functional sensor systems, such as phased-array radar, offer additional operational modes, thus requiring more intelligent strategies of sensor management that provide feedback to the process of information acquisition via appropriate control or correction commands. By this, the surveillance objectives can often be reached much more efficiently.
6. Context information is given, for example, by available knowledge on sensor and object properties, which is often quantitatively described by statistical models. Context knowledge is also given by environmental information on roads or topographical occlusions and provided by Geographical Information Systems (GIS). Seen from a different perspective, context information, such as road maps, can also be extracted from real-time sensor data directly.
7. Militarily relevant context knowledge (e.g. doctrines, planning data, tactics) and human observer reports (HUMINT: Human Intelligence) is also important information in the fusion process. The exploitation of context information of this kind can significantly improve the fusion system performance.

1.2.2 Remarks on the Methods Used

Situation elements for producing timely situation pictures are provided by integratively and spatio-temporally processing various pieces of information that in themselves often may have only limited value for understanding the situation. Essentially, logical cross-references, inherent complementarity, and redundancy are exploited. More concretely speaking, the methods used are characterized by a stochastic approach (estimating relevant state quantities) and a more heuristically defined knowledge-based approach (modeling actual human behavior when exploiting information).

Among the data exploitation products of data fusion systems, object 'tracks' are of particular importance. Tracking faces an omnipresent aspect in every real-world application insofar as it is dealing with fusion of data produced at *different instants of time*; i.e. tracking is important in all applications where particular emphasis is placed on the fact that the sensor data to be exploited have the character of a time series.

Tracks thus represent currently available knowledge on relevant, time-varying quantities characterizing the instantaneous "state" of individual targets or target groups of interest, such as aircraft, ships, submarines, vehicles, or moving persons. Quantitative measures that reliably describe the quality of this knowledge are an integral part of a track. The information obtained by 'tracking' algorithms [26, 25, 58] also includes the history of the targets. If possible, a one-to-one association between the target trajectories in the sensors' field of view and the produced tracks is to be established and has to be preserved as long as possible (track continuity). The achievable track quality does not only depend on the performance of the sensors used, but also on

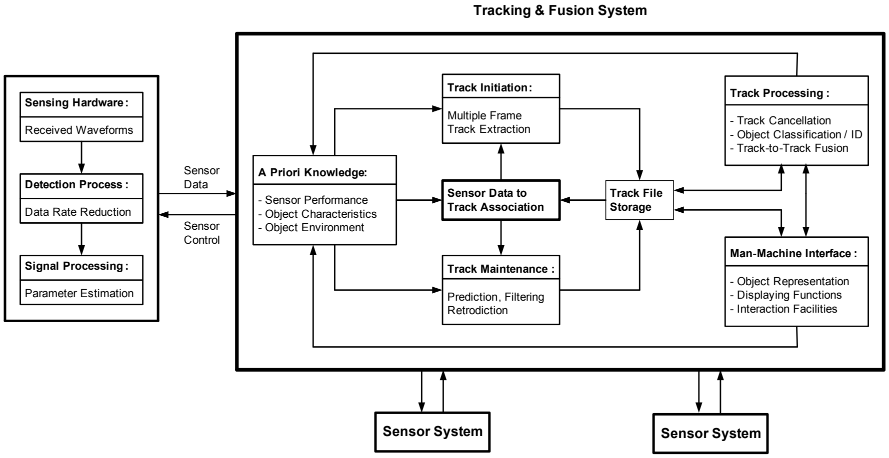

Fig. 1.4. Generic scheme of functional building blocks within a tracking/fusion system along with its relation to the sensors (centralized configuration, type IV according to O. Drummond).

target properties and the operational conditions within the scenario to be observed. If tracks 'match' with the underlying real situation within the bounds defined by inherent quality measures being part of them, we speak of 'track consistency".

Tracking algorithms, including Bayesian multiple hypothesis trackers as particularly well-understood examples, are iterative updating schemes for conditional probability density functions representing all available knowledge on the kinematic state of the objects to be tracked at discrete instants of time t_l. The probability densities are conditioned by both, the sensor data accumulated up to some time t_k, typically the current data acquisition time, as well as by available context information, such as on sensor characteristics, the object dynamics, the environment, topographical maps, or on certain rules governing the object behavior. Depending on the time instant t_l at which estimates for the state $\mathbf{x}_l$ are required, the related estimation process is referred to as prediction ($t_l > t_k$), filtering ($t_l = t_k$), or retrodiction ($t_l < t_k$) [59, 60].

1.2.3 A Generic Sensor Data Fusion System

Figure 1.4 shows a generic scheme of functional building blocks within a multiple sensor tracking and data fusion system along with its relation to the underlying sensors. In the case of multi-functional sensors, there is feedback from the tracking system to the process of sensor data acquisition (sensor management). The following aspects should be emphasized:

Sensor Systems

After passing a detection process, essentially working as a means of data rate reduction, the signal processing provides estimates of parameters characterizing the waveforms received at the sensors' front ends (e.g. radar antennas). From these estimates sensor reports are created, i.e. measured quantities possibly related to objects of interest, which are the input for the tracking and sensor data fusion system. By using multiple sensors instead of one single sensor, among other benefits, the reliability and robustness of the entire system is usually increased, since malfunctions are recognized easier and earlier and often can be compensated without risking a total system breakdown.

Interoperability

A prerequisite of all further processing steps, which at first sight seems to be trivial, is technical interoperability. It guarantees that all relevant sensor data are transmitted properly, in a timely way, and completely including all necessary meta-data describing the sensor performance, the platform parameters, and environmental characteristics. This type of meta data is necessary to transform the sensor data into common frames of reference, to identify identical pieces of data, and to merge similar pieces of data into one single augmented piece of information. The process of combining data from different sources and providing the user with a unified view of these data is sometimes also referred to as data integration. Often interoperability acts as a bottleneck in designing real-world data fusion systems of systems [4, Chapter 20].

Fusion Process

All sensor data that can be associated to existing tracks are used for track maintenance (using, e.g., prediction, filtering, and retrodiction). The remaining data are processed for initiating new tentative tracks (multiple frame track extraction). Association techniques thus play a key role in tracking/fusion applications. Context information in terms of statistical models (sensor performance, object characteristics, object environment) is a prerequisite for track maintenance and initiation. Track confirmation/termination, classification/identification, and fusion of tracks related to the same objects or object groups are part of the track management functionalities.

Human-Machine Interface

The scheme is completed by a human-machine interface with display and interaction functions. Context information can be updated or modified by direct human interaction or by the track processor itself, for example as a consequence of object classification or road map extraction. For an introduction to the vast literature on the related problems in human factors engineering and on practical systems solutions see [5].

1.2.4 On Measuring Fusion Performance

In sensor data fusion, the underlying ‘real’ situation is typically unknown. Only in expensive and time-consuming experiments certain aspects of a dynamically evolving situation are monitored, sometimes even with questionable accuracy. For this reason, experiments are valuable for demonstrating the “proof of concept” as well as to understand the underlying physical phenomena and operational problems, for example. They are of limited use, however, in performance evaluation and prediction. This underlines the role of comprehensive Monte-Carlo-simulations in fusion system performance evaluation.

According to the previous discussion, sensor data fusion systems try to establish one-to-one relations between objects in the sensors’ fields of view and identified object tracks in the situation picture. Strictly speaking, this is only possible under ideal conditions regarding the sensor performance and the underlying target scenario. It seems thus reasonable to measure the performance of a given tracking/fusion system by its characteristic deficiencies when compared to this ideal goal. In general, two categories of deficiencies can be distinguished that are either caused by mis-match regarding the input data or by non-optimal processing and unfavorable application constraints.

Selected Performance Measures

Selected performance measures or ‘measures of deficiency’ in the sense of the previous discussion, which have practical relevance in fusion systems design should be emphasized in the following.

1. Usually a time delay is involved until a track has been extracted from the sensor data. A corresponding performance measure is thus given by the ‘extraction delay’ between the first detection of a target by a sensor and a confirmed track.
2. False tracks, i.e. tracks related to unreal or unwanted targets, are unavoidable in the case of a high false return density (e.g. by clutter, jamming/deception). Corresponding ‘deficiencies’ are: mean number of falsely extracted targets per time and mean life time of a false track before its deletion.
3. Targets should be represented by one and the same track until leaving the field of view. Related performance measures are: mean life time of true target tracks, probability of an ‘identity switch’, and probability of a target not being represented by a track.
4. The track inaccuracy (given by the error covariance matrix of a state estimate, e.g.) should be as small as possible. Furthermore, the deviations between the estimated and actual target characteristics should correspond with the error covariance matrices produced (consistency). If this is not the case, ‘track loss’ usually occurs.

In a given application it is by no means simple to achieve a reasonable compromise between the various, competing performance measures and the user requirements. Optimization with respect to one measure may easily degrade other performance measures, finally deteriorating the entire system performance. This is especially true under more challenging conditions.

1.2.5 Tracking-derived Situation Elements

The primary objective of multiple sensor target tracking is to explore the underlying target kinematics such as position, velocity, or acceleration. In other words, standard target tracking applications gain information related to 'Level 1 Fusion' according to the well-established terminology of the JDL model of information fusion (see e.g. [1, Chapter 2] and the literature cited therein). Kinematic data of this type, however, are by no means the only information to be derived from target tracks. In many cases, reliable and quantitative higher level information according to the JDL terminology can be obtained. To be more concrete, wide-area air and ground surveillance is considered here as an important real-world example serving as a paradigm for other challenging tracking and fusion applications.

Inferences based on Retrodicted Tracks

The first type of higher JDL level information to be inferred from tracking data is based on a closer analysis of the histories of the kinematic object states provided by retrodiction techniques. The statements derived typically refer to object characteristics that are either time invariant or change with time on a much larger scale than kinematics quantities usually tend to do. This is the main reason why the gain in accuracy achievable by retrodiction techniques can be exploited.

- *Velocity History.* The analysis of precisely retrodicted velocity histories enables the distinction of objects belonging to different classes such as moving persons, boats, vehicles, vessels, helicopters, or jet aircraft. If the object speed estimated with sufficiently high accuracy has exceeded a certain threshold, certain object classes can be reliably be excluded. As an example, uncertainty whether an object is a helicopter or a wing aircraft can be resolved if in the track history a velocity vector 'Zero' exists. Depending on the context of the underlying application, classifications of this type can be essential to generate an alert report.
- *Acceleration History.* Similar considerations are valid if acceleration histories are taken into account: High normal accelerations, e.g., are a clear indication of a fighter aircraft. Moreover, one can safely conclude that a fighter aircraft observed with a normal acceleration $> 6\,g$, for example, is not carrying a certain type of weaponry (any more). In other words, conclusions on the threat level connected with the objects observed can be drawn by analyzing kinematic tracks.

- *Heading, Aspect Angle.* Precise reconstructions of the targets' heading vectors are not only important input information for threat evaluation and weapon assignment in themselves, but also enable estimates of the aspect angle of an object at a given instant of time with respect to other sensors, such as those producing high range or Doppler resolution spectra. Track-derived information of this type is basic for fusing spectra distributed in time and can greatly improve object classification thus providing higher-JDL-level information.
- *Rare Event Detection.* Analysis of JDL-level-1 tracks can be the key to detecting rare or anomalous events by fusing kinematic tracks with other context information such as annotated digital road maps and general rules of behavior. A simple example in the area of continuous-time, wide-area ground surveillance can be the production of an alert message if a large freight vehicle is observed at an unusual time on a dirt road in a forest region. There are analogous examples in the maritime or air domain.

Inferences based on Multiple Target Tracking

A second type of higher JDL level information related to mutual object interrelations can be inferred from JDL level 1 tracking data if emphasis is placed on the results of *multiple target* tracking.

- *Common History.* Multiple target tracking methods can identify whether a set of targets belongs to the same collectively moving group, such as an aircraft formation or a vehicle convoy, whose spatial extension may be estimated and tracked. If an aircraft formation has split off after a phase of penetration, e.g., the interrelation between the individual objects is to be preserved and provides valuable higher-JDL-level information that is important, e.g., when a former group target is classified as 'hostile' since this implies that all other targets originally belonging to the same group are likely to be hostile as well.
- *Object Sources and Sinks.* The analysis of large amounts of target tracks furthermore enables the recognition of sources and sinks of moving targets. By this type of reasoning, certain areas can be identified as air fields, for example, or an area of concentration of military forces. In combination with available context information, the analysis of multiple object tracks can also be used for target classification by origin or destination. A classification as hostile or suspect directly leads to an alert report.
- *Split-off Events.* By exploiting multiple target tracking techniques, certain split-off events can be identified as launches of air-to-air or air-to-surface missiles. The recognition of such an event from JDL-level-1 tracking information not only has implications on classifying the original target as a fighter aircraft, but can also establish a certain type of 'book-keeping', such as counting the number of missile launches. This enables estimates of the residual combat strength of the object, which has direct implications on countermeasures, e.g.

- *Stopping Events.* In the case of MTI radar (Moving Target Indicator), Doppler blindness can be used to detect the event 'A target under track has stopped.', provided this phenomenon is described by appropriate sensor models. If there is previous evidence for a missile launcher, e.g., missing data due to Doppler blindness may indicate preparation for launch with implications on potential countermeasures. In combination with other tracks, a stopping event may also establish new object interrelations, for example, when a target is waiting for another and then moving with it.

1.2.6 Selected Issues in Anomaly Detection

Anomaly detection can be regarded as a process of information fusion that combines incomplete and imperfect pieces of mutually complementary sensor data and context information in such a way that the attention of human decision makers or decision making systems is focused on particular events that are "irregular" or may cause harm and thus require special actions, such as exploiting more specialized sensors or initiating appropriate activities by military or security personnel [41]. Fusion-based anomaly detection thus improves situational awareness. What is actually meant by "regular" or "irregular" events is higher-level information itself that depends on the context of the underlying application. Here, it is either assumed to be a priori known or to be learned from statistical long-time analysis of typical situations.

In complex surveillance applications, we can often take advantage of context information on the sensing environment insofar as it is the stationary or slowly changing "stage" where a dynamic scenario evolves. Typical examples of such environmental information are digital road or sea-/air-lane maps and related information, which can essentially be regarded as spatial motion constraints (see Figure 1.5 as an illustration). In principle, this information is available by Geographical Information Systems (GIS). Another category of context information is provided by visibility models and littoral or weather maps indicating regions, where a high clutter background is to be taken into account, for example. Moreover, rather detailed planning information is often available. This category of information is not only important in mission planning or in the deployment and management of sensor systems, but can be used to decide whether an object is moving on a lane or leaving it, for example. In addition, ground-, sea- or air-lane information information can be used to improve the track accuracy of lane-moving vehicles and enhance track continuity.

Integration of Planning Information

In certain applications, rather detailed planning information is available, which provides valuable context knowledge on the temporal evolution of the objects involved and can in principle be incorporated into the tracking formalism. Planning information is often approximately described by space-time

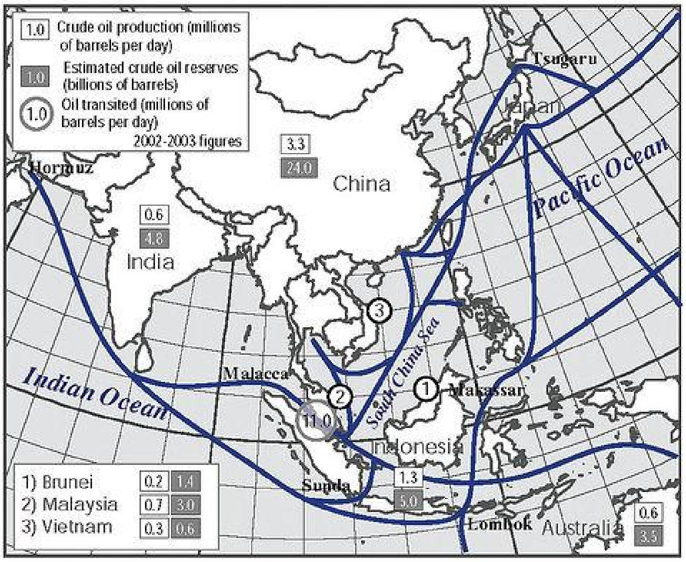

Fig. 1.5. Illustration of sea lanes and strategic passages in Pacific Asia.

waypoints that have to be passed by the individual objects during a pre-planned operation, i.e. by a set of position vectors to be reached at given instants of time and possibly via particular routes (roads, lanes) between the waypoints. In addition, we assume that the acceptable tolerances related to the arrival of the objects at the waypoints are characterized by known error covariance matrices, possibly individually chosen for each waypoint and object, and that the association between the waypoínts and the objects is predefined.

The impact of waypoints on the trajectory to be estimated from future sensor data (under the assumption that the plan is actually kept) can simply be obtained by processing the waypoints as additional artificial ‘measurements’ via the standard Bayesian tracking paradigm, where the tolerance covariance matrices are taken into account as the corresponding ‘measurement error covariances’. If this is done, the processing of sensor measurements with a younger time stamp are to be treated as “out-of sequence” measurements with respect to the artificial waypoint measurements processed earlier. According to these considerations, planning information can well improve both track accuracy and continuity as well as facilitate the sensor-data-to-track association problems involved, provided the plan is actually kept.

Detecting Regularity Pattern Violation

A practically important class of anomalies results from a violation of regularity patterns such as those previously discussed (motion on ground-, sea-, or air-lanes or following preplanned waypoints and routes). An anomaly detector thus has to decide between two alternatives:

- The observed objects obey an underlying pattern.
- The pattern is not obeyed (e.g. off-lane, unplanned).

Decisions of this type are characterized by decision errors of first and second. In most cases, it is desirable to make the decisions between both alternatives for given decision errors to be accepted. A "sequential likelihood ratio" test fulfills this requirement and has enormous practical importance. As soon as the test decided that the pattern is obeyed, the calculation of the likelihood ratio can be restarted since it is more or less a by-product of track maintenance. The output of subsequent sequential ratio tests can serve to re-confirm "normality" or to detect a violation of the pattern at last. The most important theoretical result on sequential likelihood ratio tests is the fact that the test has a *minimum decision length on average* given predefined statistical decision errors of first and second kind.

Tracking-derived Regularity Patterns

We have discussed moving targets that obey certain space-time constraints that are a priori known (roads/lanes, planned waypoints). A violation of these constraints was quite naturally interpreted as an anomaly. Seen from a different perspective, however, moving targets that are assumed to obey a priori *unknown* space-time constraints and to be observed by wide-area sensors, such as vehicles on an unknown road network, produce large data streams that can also be used for extracting the underlying space-time constraint, e.g. a road map. After a suitable post-processing, the produced tracks of motion-constrained targets simply define the corresponding constraints and can thus be extracted from tracking-based results. Extracted road-maps can be highly up-to-date and precise. A discussion where such ideas are used in wide-area maritime surveillance using AIS data can be found in [42] (AIS: Automatic Identification System).

2.0 INTEGRATION OF ADVANCED SENSOR PROPERTIES

Advanced signal processing techniques exploit even sophisticated physical phenomena of objects of interest and are fundamental to modern sensor system design. In particular, they have a direct impact on the quantitative and qualitative properties of the sensor data produced and to be fused. This makes a more subtle modeling of the statistical characteristics of the sensor output inevitable. Via constructing appropriate sensor models based on a deeper insight into the physical and technical sensor design principles, the performance of tracking and sensor data fusion systems can be significantly improved.

Chapter 2 is focused on selected physical and technical properties of sensor systems that are used in real-world ISR applications (Intelligence, Surveillance, and Reconnaissance), such as those discussed in [4, Chapter 20]. The analysis of characteristic examples shows that context information on particular performance features of the sensor systems involved is useful, in some cases even inevitable, to fulfill an overall ISR task. The Bayesian methodology discussed in Part I is wide and flexible enough to integrate more sophisticated, appropriately designed, but still mathematically tractable likelihood functions into the process of Bayesian Knowledge Propagation. The discussed examples cover finite sensor resolution, Doppler blindness, and main-lobe jamming.

The possibility to exploit even *negative sensor evidence* is a consequence that is directly connected with the use of more advanced sensor models. This notion covers the conclusions to be drawn from expected, but actually missing sensor measurements for improving the state estimates of objects under track. Even a failed attempt to detect an object of interest is a useful sensor output that is interpretable only if a consistent sensor modeling is available.

2.1 Finite Sensor Resolution

Air surveillance in a dense object / dense clutter environment is a difficult task that requires refined data association and tracking techniques. In this context, tracking for maneuvering groups of objects that join, operate closely-spaced for a while, and split off again is confronted with mainly three problems:

1. *Sensor Resolution:* Due to the limited resolution of every radar sensor, closely-spaced targets will continuously transition from being resolved to irrresolved and back again. The importance of resolution phenomena has been addressed in [125].
2. *Data Association:* Ambiguous data-to-object associations due to overlapping expectation gates are an inherent problem for formations, which is made even more difficult by high false return densities and missed detections.
3. *Maneuvers:* Often distinct maneuvering phases can be identified, as even agile objects will not always make use of their maneuvering capability. Nevertheless, abrupt turns may occur, e.g. if a formation dissolves into well-separated objects.

These problems require the use of multiple hypothesis, multiple model tracking methods as discussed in Part I. The multiple hypothesis character mirrors the uncertain origin of the data, while the multiple models refer to the different maneuvering phases. The data association problem is covered by a likelihood function $p(Z_k, m_k|\mathbf{x}_k)$ that statistically describes what a set of m_k observations $Z_k = \{\mathbf{z}_k^j\}_{j=1}^{m_k}$ can say about the joint state $\mathbf{x}_k$ of the objects to be tracked. Due to the Total Probability Theorem, it can be written as a sum over all possible, mutually exclusive, and exhaustive data interpretations j_k:

$$\begin{aligned} p(Z_k, m_k|\mathbf{x}_k) &= \sum_{j_k} p(Z_k, m_k, j_k|\mathbf{x}_k) && (2.1)\\ &= \sum_{j_k} p(Z_k|m_k, j_k, \mathbf{x}_k)\, p(m_k|j_k, \mathbf{x}_k)\, p(j_k|\mathbf{x}_k). && (2.2)\end{aligned}$$

Generally, the formulation of such likelihood functions is by no means a trivial task. In many practical cases, however, a given multiple-object tracking problem can be decomposed into independent sub-problems of reduced complexity. The example below is practically important but can still be handled more or less rigorously.

2.1.1 A Radar Resolution Model

For the sake of conciseness, we confine the discussion to non-imaging radar sensors. With some modifications, the results can also be transfered to infrared or electro-optical sensors, for example. Let us consider a medium range radar producing range and azimuth measurements of an object formation consisting of two targets. For physical reasons the resolution in range, azimuth, and range-rate will be independent from each other. In particular, range and cross-range resolution differ significantly in many radar applications. Therefore, the resolution performance of the sensor is expected to depend strongly on the current sensor-to-group geometry and the relative orientation of the targets within the group. The sensor's resolution capability is also determined by the particular signal processing techniques used and the random target fluctuations. As a complete description is rather complicated, we have to look for a simplified, but qualitatively correct and mathematically tractable model. In any case, the radar resolution capability in range and azimuth is limited by the corresponding band- and beam-width. These radar-specific parameters must explicitly enter into any processing of potentially irresolved plots. The typical size of resolution cells in a medium distance is about 50 m (range) and 500 m (cross range). As in target formations the mutual distance may well be 50 - 500 m or even less, the limited sensor resolution is a real problem in object tracking.

Centroid Measurements

Under the hypothesis $j_k = E_k^{ii}$ assuming that the radar plot $\mathbf{z}_k^i$ is an irresolved measurement belonging to two targets with a joint vector $\mathbf{x}_k = (\mathbf{x}_k^{1\top}, \mathbf{x}_k^{2\top})^\top$, the conditional likelihood is given by:

$$p(\mathbf{z}_k^i|\mathbf{x}_k) = \mathcal{N}\big(\mathbf{z}_k^i;\, \mathbf{H}_k^g\mathbf{x}_k,\, \mathbf{R}_k^g\big), \tag{2.3}$$

where the measurement matrix $\mathbf{H}_k^g$ describes a centroid measurement of the group center, characterized by a corresponding measurement error covariance matrix $\mathbf{R}_k^g$:

$$\mathbf{H}_k^g\mathbf{x}_k = \tfrac{1}{2}\mathbf{H}_k(\mathbf{x}_k^1 + \mathbf{x}_k^2), \tag{2.4}$$

where $(r_k, \varphi_k)^\top = \mathbf{H}_k\mathbf{x}_k^i$, $i = 1, 2$, is the measurement of the underlying tracking problem, where resolution phenomena are irrelevant.

Resolution Probability

Resolution phenomena will be observed if the range and angular distances between the objects are small compared with α_r, α_φ: $\Delta r_k/\alpha_r < 1$ and $\Delta\varphi_k/\alpha_\varphi < 1$. The objects within the group are resolvable if $\Delta r_k/\alpha_r \gg 1$ or $\Delta\varphi_k/\alpha_\varphi \gg 1$. Furthermore, we expect a narrow transient region. A more quantitative description is provided by introducing a resolution probability $P_r = P_r(\Delta r, \Delta\varphi)$ depending on the sensor-to-group geometry. It can be expressed by a corresponding probability of being irresolvable $P_r = 1 - P_u(\Delta r_k, \Delta\varphi_k)$. Let us describe P_u by a Gaussian-type function of the relative range and angular distances [126]:

$$P_u(\Delta r_k, \Delta\varphi_k) = \exp\left[-\log 2(\tfrac{\Delta r_k}{\alpha_r})^2\right]\, \exp\left[-\log 2(\tfrac{\Delta\varphi_k}{\alpha_\varphi})^2\right]. \tag{2.5}$$

Obviously, this simple model for describing resolution phenomena reflects the previous, more qualitative discussion. We in particular observe that P_u is reduced by a factor of 2 if Δr_k is increased from zero to α_r. Due to the Gaussian character of its dependency on the state vector $\mathbf{x}_k$ the probability P_u can formally be written in terms of a normal density:

$$P_u = \exp\left[-\log 2\,\left(\mathbf{H}_k(\mathbf{x}_k^1 - \mathbf{x}_k^2)\right)^\top \mathbf{A}^{-1}(\mathbf{H}_k\mathbf{x}_k^1 - \mathbf{H}_k\mathbf{x}_k{}^2)\right] \tag{2.6}$$

$$= \exp\left[-\log 2\,(\mathbf{H}_k^u\mathbf{x}_k)^\top \mathbf{A}^{-1}\mathbf{H}_k^u\mathbf{x}_k\right]. \tag{2.7}$$

Here the *resolution matrix* $\mathbf{A}$ is defined by $\mathbf{A} = \mathbf{diag}(\alpha_r^2, \alpha_\varphi^2)$, while the quantity $\mathbf{H}_k^u\mathbf{x}_k = \mathbf{H}_k(\mathbf{x}_k^1 - \mathbf{x}_k^2)$ can be interpreted a measurement matrix for distance measurements. Up to a constant factor the resolution probability probability $P_u(\mathbf{x}_k)$ might formally be interpreted as the fictitious likelihood function of a measurement 0 of the distance $\mathbf{H}_k(\mathbf{x}_k^1 - \mathbf{x}_k^2)$ between the objects with a corresponding *fictitious* measurement error covariance matrix $\mathbf{R}_u$ defined by the resolution parameters α_r, α_φ.

$$P_u(\mathbf{x}_k) = |2\pi\mathbf{R}_u|^{-1/2}\ \mathcal{N}\big(\mathbf{O};\, \mathbf{H}_u\mathbf{x}_k,\, \mathbf{R}_k^u\big). \tag{2.8}$$

with $\mathbf{R}_k^u = \frac{\mathbf{A}}{2\log 2} = \frac{1}{2\log 2}\mathrm{diag}[\alpha_r^2, \alpha_\varphi^2]$. According to a first order Taylor expansion, the resolution matrix describing the resolution cells in Cartesian coordinates proves to be time dependent and results from the matrix $\mathbf{A}$ by applying dilatation and a rotation. In the same way as the Cartesian measurement error ellipses, the Cartesian "resolution ellipses" depend on the target range. Suppose we have $\alpha_r = 100$ m and $\alpha_\varphi = 1°$. We then expect that the resolution in a distance of 50 km is about 100 m (range) and 900 m (cross range). Since for military targets in a formation their mutual distance may well be 200 - 500 m or even less, resolution is a real target tracking problem.

Impact of Sensor-to-Object Geometry

We expect that the resolution performance of the sensor is highly dependent on the current sensor-to-group geometry and the relative orientation of the targets within the group. As an example, let us consider the simplified situation in Figure 2.1. A formation with two targets is passing a radar. We here consider an echelon formation. R is the minimum distance of the group center from the radar.

Figure 2.2 shows the resulting probability $P_u(r; R)$ parameterized by $R = 0$, 10, 30, 60 km as a function of the distance r between the formation center and the radar. The solid lines refer to a formation approaching the radar ($\dot{r} < 0$), the dashed lines refer to $\dot{r} > 0$. For $R \neq 0$, both flight phases differ substantially. Near R, the probability P_u varies strongly ($0.85 \to 0.15$). For a radial flight ($R = 0$), we observe no asymmetry and P_u is constant over a wide range ($r \gg r_c$).

2.1.2 Resolution-specific Likelihood

For a cluster of two closely-spaced objects moving in a cluttered environment five different classes of data interpretations exist [126]:

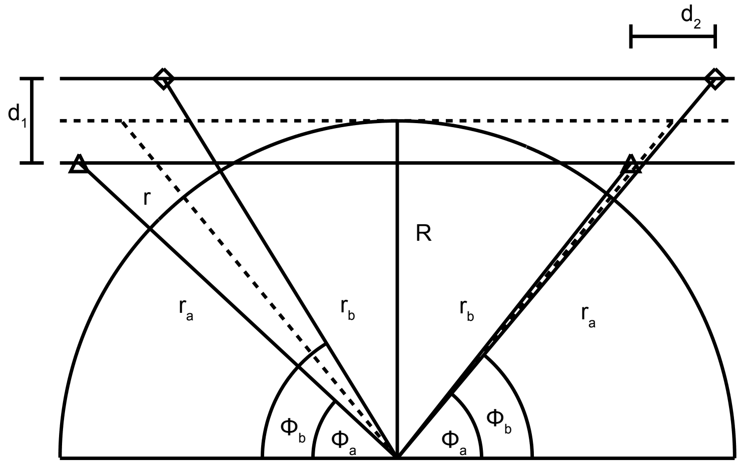

Fig. 2.1. Radar resolution phenomena: simulated object group passing a radar sensor (left: limited by azimuth resolution, right: limited by range resolution).

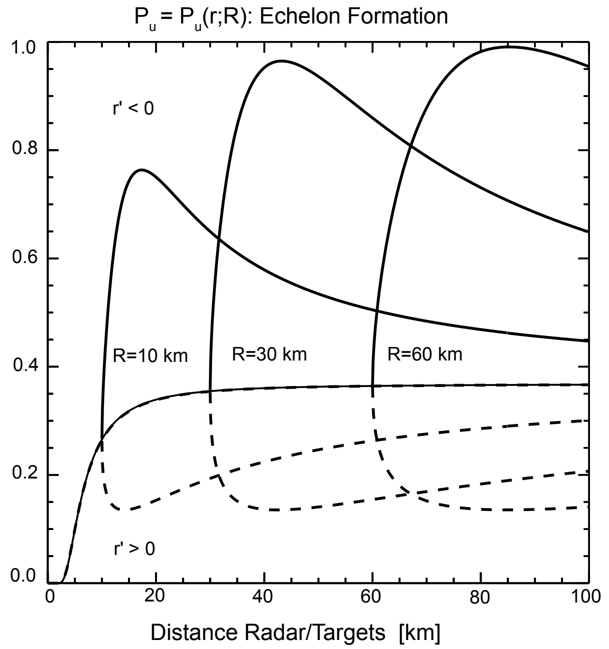

Fig. 2.2. Effect of the underlying sensor-to-group geometry: resolution probability depending on the distance between group center and radar for $R = 0$, 10, 30, 60 km.

1. E_k^{ii}, $i = 1, \ldots, m_k$: Both objects have not been resolved but detected as a group with probability P_D^u, $\mathbf{z}_k^i \in Z_k$ representing the centroid measurement; all remaining returns are false (m_k data interpretations):

   $$p(Z_k|m_k, E_k^{ii}, \mathbf{x}_k) = \frac{\mathcal{N}(\mathbf{z}_k^j; \mathbf{H}_k^g \mathbf{x}_k, \mathbf{R}_k^g)}{|\mathrm{FoV}|^{m_k-1}} \tag{2.9}$$

   $$p(m_k|E_k^{ii}, \mathbf{x}_k) = p_F(n_k - 1) \tag{2.10}$$

   $$P(E_k^{ii}|\mathbf{x}_k) = \tfrac{1}{m_k}\, P_u(\mathbf{x}_k)\, P_D^u. \tag{2.11}$$

   With P_u as represented in Equation 2.8, $p(Z_k, m_k, E_k^{ii}|\mathbf{x}_k)$ is up to a constant factor given by:

   $$p(Z_k, m_k, E_k^{ii}|\mathbf{x}_k) \propto \mathcal{N}\Big(\begin{pmatrix} \mathbf{z}_k^i \\ 0 \end{pmatrix}; \begin{pmatrix} \mathbf{H}_k^g \\ \mathbf{H}_k^u \end{pmatrix} \mathbf{x}_k, \begin{pmatrix} \mathbf{R}_k^g & \mathbf{O} \\ \mathbf{O} & \mathbf{R}_k^u \end{pmatrix} \Big). \tag{2.12}$$

   Hence, under the hypothesis E_k^{ii} two measurements are to be processed: the (real) plot $\mathbf{z}_k^i$ of the group center $\mathbf{H}_k^g \mathbf{x}_k = \frac{1}{2}\mathbf{H}_k(\mathbf{x}_k^1 + \mathbf{x}_k^2)$ and a (fictitious) measurement 'zero' of the distance $\mathbf{H}_k^u \mathbf{x}_k = \mathbf{H}_k(\mathbf{x}_k^1 - \mathbf{x}_k^2)$ between the objects. We can thus speak of *'negative' sensor information* [131], as the lack of a second target measurement conveys information on the target position. In the case of a resolution conflict, the relative target distance must be smaller than the resolution.
2. E_k^0: Both objects were neither resolved nor detected as a group, so all returns in Z_k are thus assumed to be false (one interpretation hypothesis):

   $$p(Z_k, m_k|E_k^0, x_k) = P_u(\mathbf{x}_k)\,(1 - P_D^u)\, p_F(m_k) \tag{2.13}$$

   $$P(E_k^0|\mathbf{x}_k) = P_u(\mathbf{x}_k)\,(1 - P_D^u). \tag{2.14}$$

   In analogy to the previous considerations, we can write up to a constant factor:

   $$p(Z_k, m_k, E_k^0|\mathbf{x}_k) \propto \mathcal{N}\big(0;\, \mathbf{H}_k^u \mathbf{x},\, \mathbf{R}_k^u\big). \tag{2.15}$$

   This means that even under the hypothesis of a missing irresolved plot, at least a fictitious distance measurement 0 is processed with a measurement error given by the sensor resolution.
3. E_k^{ij}, $i, j = 1, \ldots, m_k$, $i \neq j$: Both objects were resolved and detected, $\mathbf{z}_k^i, z_k^j \in Z_k$ are the measurements, $m_k - 2$ returns are false ($m_k(m_k - 1)$ interpretations):

   $$p(Z_k|m_k, E_k^{ij}, \mathbf{x}_k) = \frac{\mathcal{N}(\mathbf{z}_k^i; \mathbf{H}_k \mathbf{x}_k^1, \mathbf{R}_k)\, \mathcal{N}(\mathbf{z}_k^j; \mathbf{H}_k \mathbf{x}_k^2, \mathbf{R}_k)}{|\mathrm{FoV}|^{m_k-2}} \tag{2.16}$$

   $$p(m_k|E_k^{ij}, \mathbf{x}_k) = p_F(m_k - 2) \tag{2.17}$$

   $$P(E_k^{ij}|\mathbf{x}_k) = \tfrac{\big(1 - P_u(\mathbf{x}_k)\big)}{m_k(m_k-1)}\, P_D^2. \tag{2.18}$$

According to the factor $1 - P_u(\mathbf{x}_k) = 1 - |2\pi\mathbf{R}_u|^{\frac{1}{2}}\ \mathcal{N}\big(0;\, \mathbf{H}^u_k\mathbf{x},\, \mathbf{R}^u_k\big)$ the likelihood function becomes a mixture, in which *negative* weighting factors can occur. Nevertheless, the coefficients sum up to one; the density $p(\mathbf{x}_k|Z^k)$ is thus well-defined. This reflects the fact that in case of a resolved group the targets must have a certain minimum distance between each other which is given by the sensor resolution. Otherwise they would not have been resolvable.

4. E^{i0}_k, E^{0i}_k, $i = 1, \ldots, m_k$: Both objects were resolved but only one object was detected, $\mathbf{z}^i_k \in Z_k$ is the measurement, $m_k - 1$ returns in Z_k are false ($2m_k$ interpretations):

$$p(Z_k, m_k|E^{i0}_k, \mathbf{x}_k) = |\mathrm{FoV}|^{1-n_k}\ \mathcal{N}(\mathbf{z}^i_k;\, \mathbf{H}_k\mathbf{x}^1_k,\, \mathbf{R}_k)\, p_F(m_k - 1) \tag{2.19}$$

$$P(E^{i0}_k|\mathbf{x}_k) = \tfrac{1}{m_k}\,\big(1 - P_u(\mathbf{x}_k)\big)\, P_D\,(1 - P_D). \tag{2.20}$$

5. E^{00}_k: The objects were resolved, but not detected; all m_k plots in Z_k are false (one interpretation):

$$p(Z_k, m_k|E^{00}_k, x_k) = |\mathrm{FoV}|^{-m_k}\ p_F(m_k) \tag{2.21}$$

$$P(E^{00}_k|\mathbf{x}_k) = \big(1 - P_u(\mathbf{x}_k)\big)\,(1 - P_D)^2. \tag{2.22}$$

Since there exist $(m_k + 1)^2 + 1$ interpretation hypotheses, the ambiguity for even small clusters of closely-spaced objects is much higher than in the case of well-separated objects ($m_k + 1$ each). This means that only small groups can be handled more or less rigorously. For larger clusters (raids of military aircraft, for instance) a collective treatment seems to be reasonable until the group splits off into smaller sub-clusters or individual objects.

Up to a factor $\frac{1}{m_k!}\,\rho_F^{m_k-2}\,|FoV|^{-m_k}\,\mathrm{e}^{-|\mathrm{FoV}|\rho_F}$ independent of $\mathbf{x}_k$, the likelihood function of potentially irresolved sensor data in a clutter background,

$$p(Z_k, m_k|x_k) = p(Z_k, m_k, E^0_k) + \sum_{i,j=0}^{m_k} p(Z_k, E^{ij}_k, m_k|x_k), \tag{2.23}$$

is proportional to a sum of Gaußians and a constant:

$$\begin{aligned}
p(Z_k, n_k|\mathbf{x}_k) \propto{}& \rho_F^2(1 - P_D)^2\big(1 - P_u(\mathbf{x}_k)\big) + \rho_F(1 - P^u_D)P_u(\mathbf{x}_k) + \\
&P^u_D\rho_F P_u(\mathbf{x}_k)\sum_{i=1}^{n_k} \mathcal{N}(\mathbf{z}^i_k;\, \mathbf{H}^g_k\mathbf{x}_k,\, \mathbf{R}^g_k) + \\
&\rho_F P_D(1 - P_D)\big(1 - P_u(\mathbf{x}_k)\big)\sum_{i=1}^{n_k}\big\{\mathcal{N}(\mathbf{z}^i_k;\, \mathbf{H}_k\mathbf{x}^1_k,\, \mathbf{R}_k) + \mathcal{N}(\mathbf{z}^i_k;\, \mathbf{H}_k\mathbf{x}^2_k,\, \mathbf{R}_k)\big\} + \\
&P_D^2\big(1 - P_u(\mathbf{x}_k)\big)\sum_{\substack{i,j=1\\ i\neq j}}^{n_k} p^{ij}_k(\mathbf{x}_k)\ \mathcal{N}(\mathbf{z}^i_k;\, \mathbf{H}_k\mathbf{x}^1_k,\, \mathbf{R}_k)\ \mathcal{N}(\mathbf{z}^j_k;\, \mathbf{H}_k\mathbf{x}^2_k,\, \mathbf{R}_k).
\end{aligned} \tag{2.24}$$

2.1.3 A Formation Tracking Example

If the spatial false return density is not too high, JPDA-type approximations [25] can be applied. According to this philosophy, the joint state mixture density $p(\mathbf{x}_k^1, \mathbf{x}_k^2 | \mathcal{Z}^k)$ resulting from the likelihood function previously discussed is approximated by a single Gaussian with the same expectation vector and covariance matrix as the mixture $p(\mathbf{x}_k^1, \mathbf{x}_k^2 | \mathcal{Z}^k)$ (moment matching [25, p. 56 ff]). Objects moving closely-spaced for some time irreversibly lose their identity: When they dissolve again, a unique track-to-target association is impossible. It is thus reasonable to deal with densities that are symmetric under permutations of the individual targets. Thus, no statistically relevant information is lost and the filter performance remains unchanged, while the mean number of hypotheses involved may be significantly reduced. Within the framework of JPDA-type approximations, this has the following effect: Before combining two components of the mixture via moment matching, we check if the components are more 'similar' to each other when the target indices are switched. If this is the case, we combine them instead. These considerations are also a useful and simple means to avoid track coalescence.

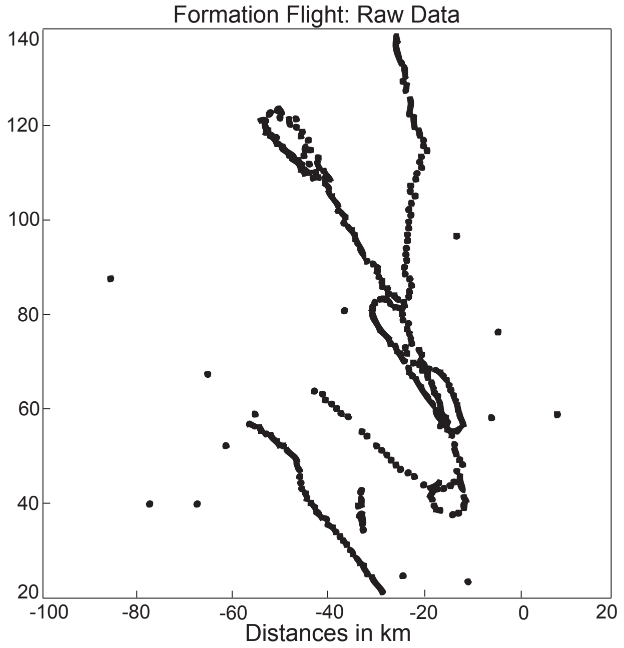

Fig. 2.3. Partly irresolved aircraft formation: accumulated raw data of a mid-range radar.

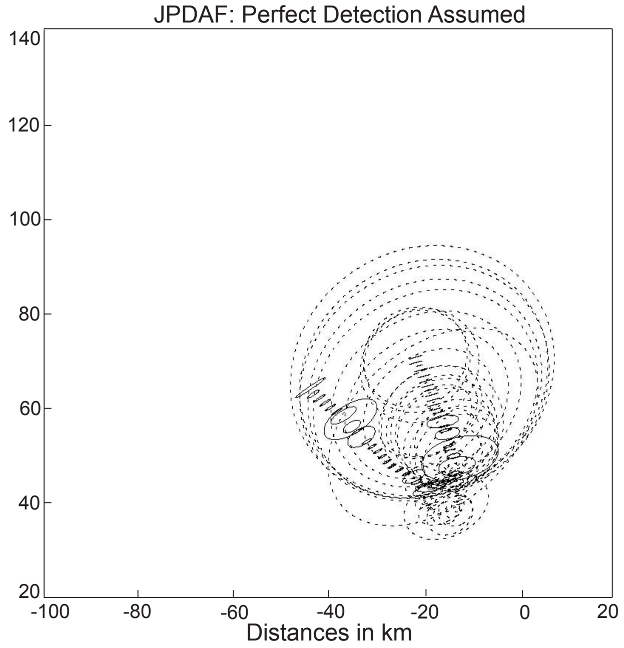

Fig. 2.4. Tracking of an aircraft formation: filtering results (JPDA, no resolution model).

Figure 2.3 shows a set of data from a typical medium-range radar. The scan interval is 5 sec and the detection probability about 80%. The example clearly shows that resolution must be taken into account as soon as the targets begin to move closely-spaced. Figures 2.4, 2.5 show the estimation error ellipses for two targets (red, white) resulting from JPDA filtering. While in Figure 2.4 perfect sensor resolution was assumed (wrongly!), in Figure 2.5 the previous resolution model was used. JPDA filtering without considering resolution phenomena evidently fails after a few frames, as indicated by diverging tracking error ellipses. This has a simple explanation: without modeling the limited sensor resolution, an actually produced irresolved plot can only be treated as a single target measurement along with a missed detection. In consequence, the related covariance matrices increase in size. This effect is further intensified by subsequent irresolved returns. If hypotheses related to resolution conflicts are taken into account, however, the tracking remains stable. The error ellipses in Figures 2.4, 2.5 have been enlarged to make their data-driven adaptivity more visible. The ellipses shrink, for instance, if both targets are actually resolved in a particular scan. The transient enlargement halfway during the formation flight is caused by a crossing target situation.

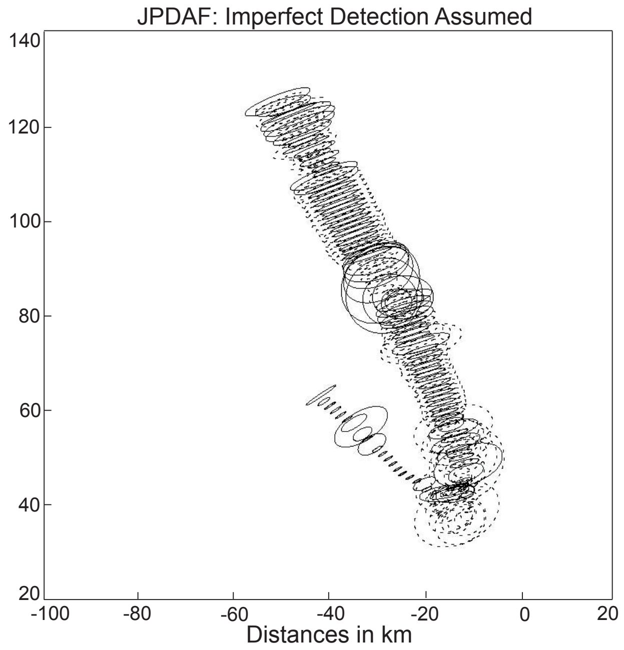

Fig. 2.5. Tracking of an aircraft formation: filtering results (with resolution model).

2.1.4 Resolution: Summary of Results

MHT filtering with explicit handling of resolution conflicts can successfully be applied to real radar data [69]. The main conclusions of extensive simulations based on exemplary scenarios and typical radar parameters are [122]:

1. For objects with overlapping expectation gates and potentially irresolved measurements, MHT filters that handle data association conflicts in combination with resolution phenomena by far outperform more conventional trackers (monohypothesis approximations or filters ignoring imperfect resolution). Much higher false return densities and significantly lower detection probabilities can be tolerated, the tracks are more accurate, the correlation gates are reduced in size, and the critical phases of joining and splitting-off are supported.
2. Provided only primary radar data are available, information on the object identity rapidly fades out while the objects move closely-spaced and produce potentially irresolved plots. After splitting off again, a unique track-to-target correlation is no longer possible. We may thus drop the notion of identity and deal with indistinguishable targets. By this, no statistically relevant information is lost, i.e. the number of hypotheses involved can significantly be reduced without affecting the track accuracy.

3. Whether an object group is resolvable or not is highly dependent on the specific sensor-to-object geometry considered and on the position of the objects relative to each other. This phenomenon is adaptively taken into account by the resolution model used. As the correct association hypotheses can reliably be reconstructed by retrodiction techniques at the expense of some delay, the resolution model may in a retrospective view provide information on the relative position of the targets within the formation.
4. Besides the ambiguity due to irresolved or missed detections, overlapping correlation gates, and false returns, scenarios with highly maneuvering targets are also ambiguous with respect to the object evolution model assumed to be in effect. Hypotheses related to resolution conflicts fit well into the more complex framework of IMM-MHT and provide performance improvements over more simplified dynamics models.

Key Publication

A detailed discussion of this approach has been published in:

- W. Koch and G. van Keuk

  Multiple Hypothesis Track Maintenance with Possibly Unresolved Measurements

  IEEE Transactions on Aerospace and Electronic Systems, Vol. 33, No. 3, p.883-892, July 1997.
  An extended version with results from various related conference papers of the author has been published as a handbook chapter in: W. Koch. Target Tracking. Chapter 8 in: *Stergios Stergiopoulos (Ed.). Advanced Signal Processing: Theory and Implementation for Sonar, Radar, and Non-Invasive Medical Diagnostic Systems.* CRC Press (2001).

  Abstract
  In surveillance problems, dense clutter/dense target situations call for refined data association and tracking techniques. In addition, closely-spaced targets may exist which are not resolved. This phenomenon has to be considered explicitly in the tracking algorithm. We concentrate on two targets that temporarily move in close formation and derive a generalization of MHT methods on the basis of a simple resolution model.

  Key words: sensor resolution, Bayesian multiple target tracking, multiple hypothesis tracking, target formations

2.2 GMTI Radar: Doppler Blindness

Ground surveillance comprises track extraction and maintenance of single ground-moving vehicles and convoys, as well as low-flying objects such as helicopters or Unmanned Aerial Vehicles. As ground object tracking is a challenging problem, all available information sources must be exploited, i.e. the sensor data themselves, as well as context knowledge about the sensor performance and the underlying scenario.

2.2.1 Air-to-Ground Surveillance

For long-range, wide-area, all-weather, and all-day surveillance operating at high data update rates, GMTI radar proves to be the sensor system of choice (GMTI: Ground Moving Target Indication). By using airborne sensor platforms in stand-off ground surveillance applications, the effect of topographical screening is alleviated, thus extending the sensors' field of view. In [144] characteristic problems of signal processing related to GMTI tracking with STAP radar are discussed. In this context, the following topics are of particular interest:

- *Doppler-Blindness.* Ground moving vehicles can well be masked by the clutter notch of the sensor. This physical phenomenon directly results from the low-Doppler characteristics of ground-moving vehicles and causes interfering fading effects that seriously affect track accuracy and track continuity. The problems are even more challanging in the presence of Doppler ambiguities.
- *Collectively Moving Targets.* Collectively moving convoys consisting of individual vehicles are typical of certain applications and have to be treated as aggregated entities. In some cases, the kinematic states of the individual vehicles can be treated as internal degrees of freedom. In addition, the convoy extension can become part of the object state.
- *Road-Map Information.* Even military targets usually move on road networks, whose topographical coordinates are known in many cases. Digitized topographical road maps such as provided by Geographical Information Systems (GIS) should therefore enter into the target tracking and sensor data fusion process.
- *Multisensor Data.* Since a single GMTI sensor on a moving airborne platform can record a situation of interest only over short periods of time, sensor data fusion proves to be of particular importance. The data processing and fusion algorithms used for ground surveillance are closely related to the statistical, logical, and combinatorial methods applied to air surveillance.

2.2.2 A Model for Doppler Blindness

For physical and technical reasons, the detection of ground-moving targets by airborne radar, typically on a moving platform, is limited by strong ground clutter returns. This can be much alleviated by STAP techniques [144]. The characteristics of STAP processing, however, directly influence the GMTI tracking performance. Even after platform motion compensation by STAP filtering low-Doppler targets can be masked by the clutter notch of the GMTI radar. Let $\mathbf{e}_k^p = (\mathbf{r}_k - \mathbf{p}_k)/|\mathbf{r}_k - \mathbf{p}_k|$ denote the unit vector pointing from the platform position $\mathbf{p}_k$ at time t_k to the target at the position $\mathbf{r}_k$ moving with the velocity $\dot{\mathbf{r}}_k$. The kinematic object state is given by $\mathbf{x}_k = (\mathbf{r}_k^\top, \dot{\mathbf{r}}_k^\top)^\top$. Doppler blindness occurs if the radial velocities of the object as well as of the surrounding main-lobe clutter return are identical, i.e. if the function

$$h_n(\mathbf{r}_k, \dot{\mathbf{r}}_k; \mathbf{p}_k) = \frac{(\mathbf{r}_k - \mathbf{p}_k)^\top \dot{\mathbf{r}}_k}{|\mathbf{r}_k - \mathbf{p}_k|} \tag{2.25}$$

is close to zero. In other words, $h_c(\mathbf{x}_k; \mathbf{p}_k) \approx 0$ holds if the target's velocity vector is nearly perpendicular to the sensor-to-target line-of-sight. For this reason, the equation $h_c(\mathbf{x}_k; \mathbf{p}_k) = 0$ defines the location of the GMTI clutter notch in the state space of a ground target and as such reflects a fundamental physical/technical fact without implying any further modeling assumptions.

Qualitative Discussion

Any GMTI detection model for air-to-ground radar must thus reflect the following phenomena:

1. The detection probability P_D depends on the target state and the sensor/target geometry.
2. P_D is small in a certain region around the clutter notch characterized by the Minimum Detectable Velocity (MDV), an important sensor parameter that must enter into the tracking process.
3. Far from the clutter notch, the detection probability depends only on the directivity pattern of the sensor and the target range.
4. There exists a narrow transient region between these two domains.

GMTI models are adapted to STAP techniques in that the detection probability assumed in the tracking process is described as a function of the GMTI-specific clutter notch. While the current location of the notch is determined by the kinematical state of the target and the current sensor-to-target geometry, its width is given by a characteristic sensor parameter (MDV). In this way, more detailed information on the sensor performance can be incorporated into the tracking process. This in particular permits a more appropriate treatment of missing detections. In other words, information on the potential reasons that might have caused the missing detections enters into the tracking filter. We observed that by this measure, the number of lost tracks can significantly be reduced, while the track continuity is improved, finally leading to a more reliable ground picture. This qualitative discussion of the observed detection phenomena related to the GMTI clutter notch is similar in nature to that of resolution effects.

Quantitative Discussion

In a generic description of the detection performance of GMTI sensors it seems plausible to write $P_D = P_D(\mathbf{x}_k)$ as a product with one factor reflecting the directivity pattern and propagation effects due to the radar equation [71], $p_D = p_D(r_k, \varphi_k)$, the other factor being related to the clutter notch. To this end, let us consider functions of the following form:

$$P_D(r_k, \varphi_k, \dot{r}_k) = p_d(r_k, \varphi_k) \left(1 - \mathrm{e}^{-\frac{1}{2}\left(\frac{h_n(r_k, \varphi_k, \dot{r}_k)}{\mathrm{MDV}}\right)^2}\right). \tag{2.26}$$

In this expression the sensor parameter MDV has a clear and intuitive meaning: In the region defined by $|n_c(\mathbf{x}_k)| < \mathrm{MDV}$ we have $P_D < \frac{1}{2}\, p_d$. The parameter MDV is thus a quantitative measure of the minimum radial velocity with respect to the sensor platform that a ground-moving target must at least have to be detectable by the sensor (Minimum Detectable Velocity). The actual size of MDV depends on the particular signal processor used.

For SWERLING I targets p_d is given by: $p_d(r, \varphi) = {p_F}^{1/[1+\mathrm{snr}(\mathrm{r},\varphi)]}$ with the false alarm probability p_F and the signal-to-noise ratio $\mathrm{snr}(r, \varphi) = \mathrm{snr}_0 D(\varphi)(\sigma/\sigma_0)(r/r_0)^{-4}$ according to [71]. Let the sensor's directivity pattern be described by $D(\varphi) = \sin^2(\varphi - \varphi_a)$.

After rearranging the terms in Equation 2.26, we can formally introduce Gaußian likelihood functions, where $h_n(\mathbf{x}_k)$ appears as a fictitious nonlinear measurement function:

$$P_D(\mathbf{x}_k; \mathbf{p}_k) = P_D - P_D^n\, \mathcal{N}(0; h_n(\mathbf{x}_k; \mathbf{p}_k), R_n)\big), \tag{2.27}$$

with a detection parameter P_D^n and a related 'variance' R_n given by a function of MDV.

Impact of Sensor-to-Object Geometry

Assuming a flat earth, Figure 2.6 shows an idealized scenario with two airborne GMTI sensors observing a ground vehicle moving at a constant speed (15 m/s = 54 km/h) parallel to the x-axis for most of the time. This situation is typical of stand-off or gap-filling ground surveillance missions. In the second half of the observation period over $\Delta t_{\max} = 25$ min the target stops for 7 min. Then it speeds up again reaching its initial velocity. Finally, the target leaves the field of view of sensor 2. In Table 2.1 selected sensor and platform parameters are summarized. h_p, v_p denote the constant height and speed of the sensor platforms over ground. Δr, $\Delta\varphi$ are the range and azimuth regions covered by each sensor during observation. The revisit intervals are given by ΔT, while MDV denotes the Minimum Detectable Velocity, a GMTI-specific sensor parameter important to ground-moving target tracking. Unless appropriately handled, two phenomena in particular can cause problems in GMTI tracking:

1. Sensor-to-target geometries can occur where targets to be tracked are masked by the clutter notch of the sensor. This results in a series of missing detections until the geometry changes again.

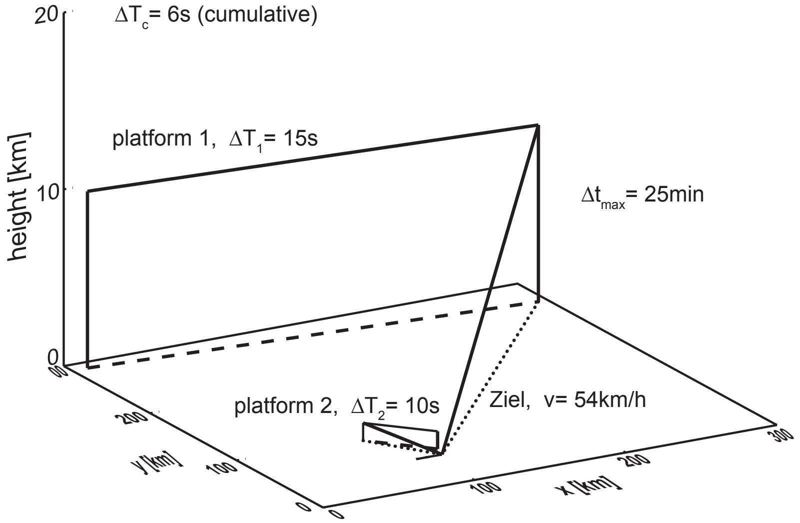

Fig. 2.6. Simplified ground target tracking scenario: two moving airborne GMTI radar platforms and a single ground moving target.

Sensor	h_p [km]	v_p [m/sec]	Δr [km]	$\Delta\varphi$ [deg]	ΔT [sec]	MDV [m/sec]
1	10	200	[232, 292]	[-128, -67]	15	2
2	1	40	[22, 54]	[77, 172]	10	2

Table 2.1. Simplified GMTI tracking scenario: selected sensor and platform parameters.

2. As stopping targets are indistinguishable from ground clutter, the early detection of a stopping event itself as well as tracking of ‘stop & go’ targets can be important to certain applications.

The impact of these effects on the detection probability is shown in Figure 2.7 for the scenario previously introduced. For both sensors we observe deep notches (dashed line: platform 1, dotted line: platform 2). In the center of these notches the radial velocities of the target and the surrounding ground patch are very close to each other, thus making target discrimination by Doppler processing (STAP [144]) impossible. This is particularly true if the target stops.

The dashed and solid lines in Figure 2.8 denote the radial velocities of ground patches around the target and target returns, respectively. The area shaded in gray reflects the width of the clutter notches of the sensors, which is determined by the individual Minimum Detectable Velocities (MDVs). For each sensor, both curves are closely adjacent to each other, indicating that the target is moving at a much lower speed than the sensor platforms. We

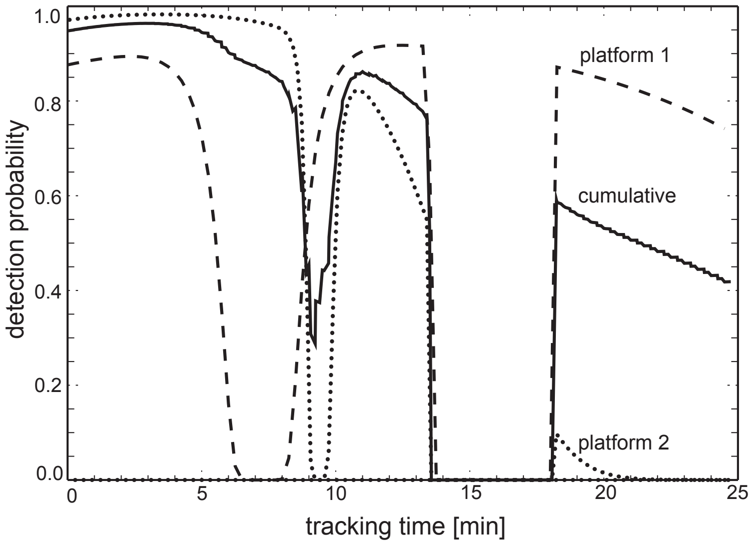

Fig. 2.7. GMTI tracking: detection probability of the individual sensors and the mean accumulated detection probability as a function of the tracking time.

notice sliding intersections between the curves. They are responsible for the relatively long duration of Doppler-blind phases.

Assuming an idealized processing architecture (measurement fusion), the *mean cumulative revisit interval* ΔT_c results from the individual revisit intervals $\Delta T_1 = 15$ s, $\Delta T_2 = 10$ s, yielding $\Delta T_c = 6$ s. The *mean cumulative detection probability* P_D^c is shown in Figure 2.7 (solid line). The impact of the clutter notches is more or less compensated for. Due to the fact that P_D^c is related to the mean cumulative revisit interval $\Delta T_c = 6$ s, being shorter than those of the individual sensors ($\Delta T_1 = 10$ s, $\Delta T_2 = 15$ s), P_D^c is smaller than the detection probability of the sensor dominating at that time.

On Convoy Resolution

Since in certain applications, ground traffic vehicles often move in convoys, at first view resolution phenomena seem to be typical of long-range ground surveillance. Due to the asymmetric effect of range and angle resolution, however, Doppler-blindness in many cases superimposes resolution effects. As soon as convoy targets cease to be resolvable, they are at the same time buried in the clutter notch and thus escape detection. Vice versa, resolvable convoy targets are rarely Doppler-screened. A separate modeling of the sensor resolution might therefore be omitted.

As an example we assume two targets moving in a row along a straight road with 30 km/h as typical of military applications. Their mutual distance is 50 m. The target/sensor geometry is as depicted in Figure 2.6. Let the sensor

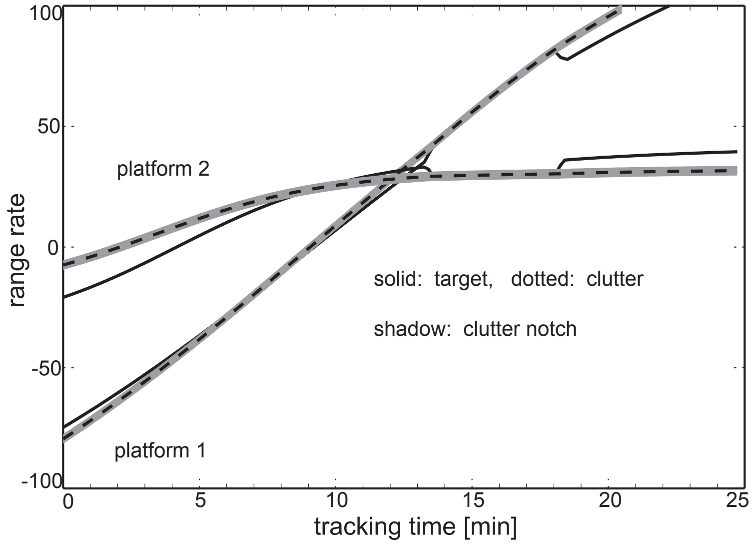

Fig. 2.8. GMTI tracking: range rate of the ground target and the surrounding ground patch relative to the moving GMTI sensors.

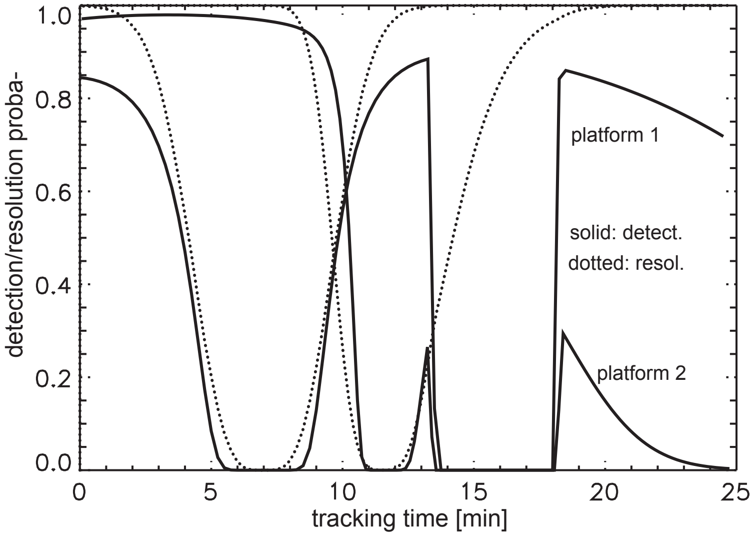

Fig. 2.9. detection and resolution probability

resolution be given by: $\alpha_r = 10$ m (range), $\alpha_\varphi = 0.1°$ (azimuth), $\alpha_{\dot{r}} = 0.5$ m/s (range-rate). Figure 2.9 shows the detection probabilities of both sensors (solid lines). The width of the notches is larger than in Figure 2.7 due to the smaller convoy speed. The dotted lines denote the resolution probabilities P_r of the sensors:

$$P_r = 1 - e^{-\log 2(\Delta r/\alpha_r)^2}\, e^{-\log 2(\Delta\varphi/\alpha_\varphi)^2}\, e^{-\log 2(\Delta\dot{r}/\alpha_{\dot{r}})^2}. \tag{2.28}$$

Δr, $\Delta\varphi$, $\Delta\dot{r}$ are the distances between the targets in sensor coordinates. If P_r is dominated by the angular resolution (i.e. Δr and $\Delta\dot{r}$ are small), Doppler-blindness occurs. Outside of the notch the high range/range-rate resolution guarantees resolved returns.

2.2.3 Essentials of GMTI Tracking

The choice of a suitable coordinate system for describing the underlying sensor/target geometry, the sensor platform trajectory, and the available a priori information on the dynamical behavior of ground-moving targets are prerequisites to target tracking. In wide-area applications a flat earth model is often not admissible. We consider three coordinate systems in which the underlying physical phenomena become transparent:

1. Appropriate *ground* coordinates, typically WGS84, where the description of the target and platform kinematics is of a particularly simple form,
2. the moving Cartesian *antenna* coordinate system, whose x-axis is oriented along the array antenna of the GMTI radar mounted on the airborne sensor platform,
3. the *sensor* coordinate system, in which the measurements of the kinematical target parameters are described (target range, azimuth, and range-rate).

Under the appropriate assumptions, the likelihood is given by the following expression (single vehicle, mild residual clutter density ρ_F, m_k plots in each sensor scan $Z_k = \{\mathbf{z}_k^j\}_{j=1}^{m_k}$):

$$\begin{aligned} p(Z_k, m_k|\mathbf{x}_k) &= \big(1 - P_D(\mathbf{x}_k;\mathbf{p}_k)\big)\rho_F \;+\; P_D(\mathbf{x}_k;\mathbf{p}_k)\textstyle\sum_{j=1}^{m_k}\mathcal{N}\big(\mathbf{x}_k;\,\mathbf{h}(\mathbf{x}_k),\,\mathbf{R}\big) \\ &= p_0(Z_k, m_k|\mathbf{x}_k) + p_n(Z_k, m_k|\mathbf{x}_k) \end{aligned} \tag{2.29}$$

where $p_0 = p_0(Z_k, m_k|\mathbf{x}_k)$ denotes the standard likelihood without considering clutter notches:

$$p_0 = (1 - P_d)\rho_F + P_d \textstyle\sum_{j=1}^{m_k}\mathcal{N}(\mathbf{x}_k;\mathbf{h}(\mathbf{x}_k),\mathbf{R}), \tag{2.30}$$

$p_n = p_n(Z_k, m_k|\mathbf{x}_k)$ is the part of the overall likelihood function characteristic of the GMTI problem. For a generalization in case of Doppler-unambiguous measurements see [67, 127].

If the GMTI detection model is inserted into this expression, we immediately see that the effect of the GMTI-specific clutter notch on the likelihood function can formally be described by a fictitious measurements Zero of a fictitious quantity defined by pseudo measurement function $\mathbf{h}_k^n$, where the minimum detectable velocity plays the role of a fictitious measurements error standard deviation.

According to Bayes' rule, the processing of the new sensor data Z_k received at revisit time t_k is based on the predicted density $p(\mathbf{x}_k|\mathcal{Z}^{k-1})$ and the likelihood function $p(Z_k, n_k|\mathbf{x}_k)$. Assuming a Gaußian sum representation for $p(\mathbf{x}_k|\mathcal{Z}^{k-1})$, the Gaußian sum structure of the likelihood function guarantees that also $p(\mathbf{x}|Z^k)$ belongs to this family. According to Bayes Theorem we obtain up to a normalizing constant:

$$p(\mathbf{x}_k|\mathcal{Z}^k) \propto p(Z_k, n_k|\mathbf{x}_k)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \tag{2.31}$$

$$\propto \sum_i p_k^i \, \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k}^i,\, \mathbf{P}_{k|k1}^i\big). \tag{2.32}$$

Mixture reduction techniques (pruning, local combining) can be applied in order to keep the number of mixture components under control. Simulations showed that even a representation by only two mixture components is sufficient in many practical cases and seems to mirror the underlying physics of the detection process quite well.

2.2.4 Effect of GMTI-Modeling

Figures 2.10 – 2.12 provide a qualitative insight into the effect of the refined sensor model on target tracking/data fusion. While a high adaptivity is evident near the clutter notch, far from the notch no difference to standard filters is observed.

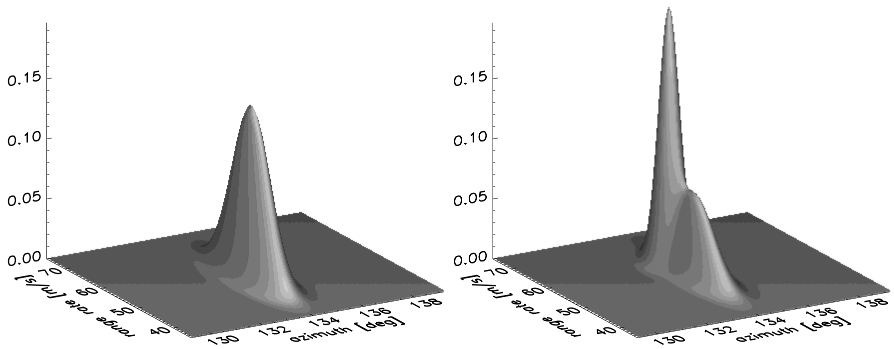

Fig. 2.10. Effect of GMTI modeling (missing detection near the clutter notch): (a) standard filter, (b) GMTI filter).

Figure 2.10 displays the probability density functions resulting from processing the event that a missing detection occurred near the notch. To show the most interesting features, the densities are projected on the azimuth/range-rate plane. While the probability density the standard tracker (Figure 2.10a) is identical with the corresponding predicted density, the refined sensor model leads to a bimodal structure (Figure 2.10b). The broader peak refers to the possible event that the missing detection has purely statistical reasons as in the case of standard filtering, while the sharper peak behind it reflects the hypothesis that the target was not detected because it is masked by the clutter notch.

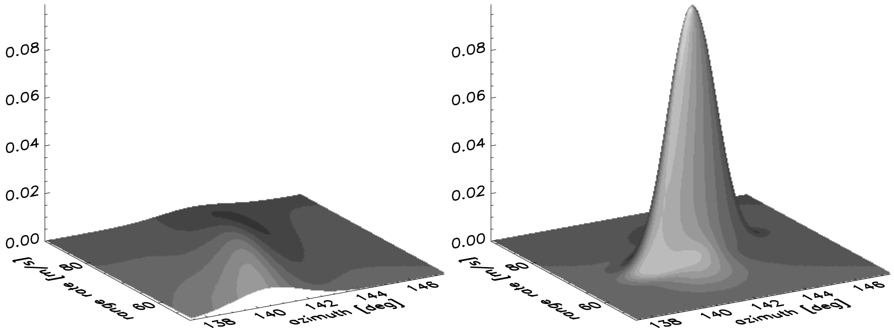

Fig. 2.11. Effect of GMTI modeling (target buried in the notch for several revisits): (a) standard filter. (b) GMTI filter.

The situation where the target is buried in the clutter notch for several revisits is represented in Figure 2.11. Obviously, the probability density of the standard filter totally faded away permitting no reasonable state estimation (Figure 2.11a). The refined filter, however, preserved a definite shape (Figure 2.11b). This can be explained as follows. Instead of actual sensor data, the very information that several successively missing detections occurred was processed. This event provides a hint to the filter that the kinematical target state probably obeys a certain relation determined by the clutter notch. Apparently, this piece of evidence proves to be as valuable as a measurement of one of the components of the target state.

Figure 2.12 refers to the event that a detection occurred near the clutter notch. While the standard filter produced a simple Gaußian, the refined filter shows a more complex structure. In fact, the probability density is a two-component mixture whose weighting factors differ in their sign (but sum up to one). The resulting shape permits an intuitive interpretation. The sensor model inherently takes into account the fact that the target state $\mathbf{x}_k$ does not lead to a small value of $n_c(\mathbf{x}_k)$; otherwise the target would not have been

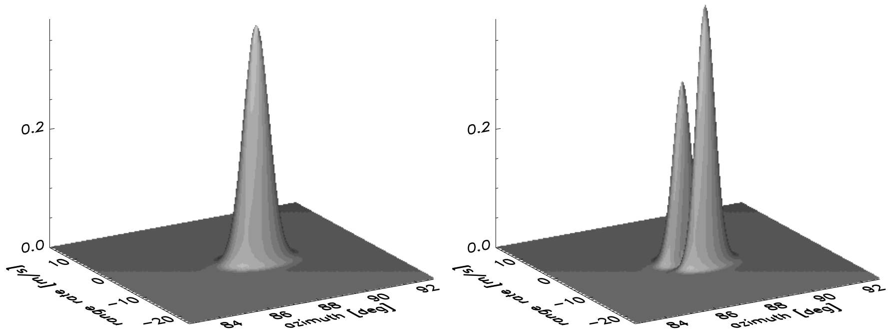

Fig. 2.12. Effect of GMTI modeling (detection occurs near the clutter notch): (a) standard filter. (b) GMTI filter.

detected at all. For this reason, the sharp cut in the probability density simply indicates the location of the clutter notch.

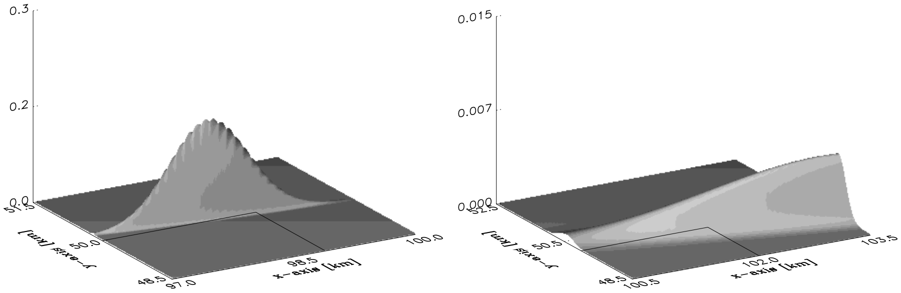

Fig. 2.13. Gain by processing GMTI data from sensor 1 only: (a) during tracking. (b) target stop.

Gain by Sensor Data Fusion

Figures 2.13 – 2.15 show the probability densities of the target position in Cartesian ground coordinates after filtering. The prolated structure of the probability densities mirrors the predominant impact of cross-range errors. Their shape is rotated with respect to each other due to the different sensor-to-target geometries. This effect can be much more pronounced in other situations. We indicated the true target position. Figures 2.13a – 2.15a refer to a regular tracking situation (after 10 min, see Figures 2.6, 2.7). Doppler-blindness occurred for sensor 2 during the previous revisits. The probability densities shown in Figure 2.13b – 2.15b have been calculated at a time when

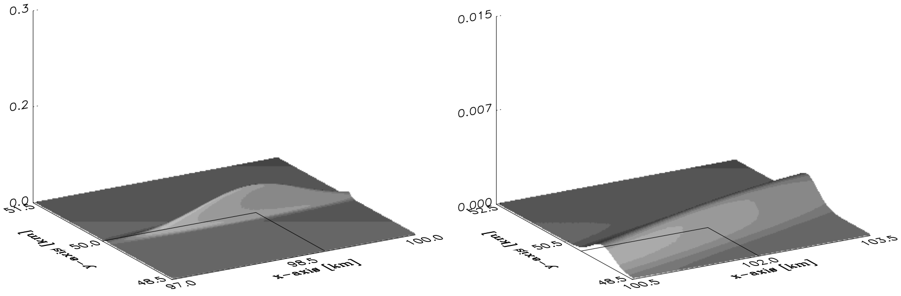

Fig. 2.14. Gain by processing GMTI data from sensor 2 only: (a) during tracking. (b) target stop.

the target has stopped for 3 min. Evidently in Figures 2.13b, 2.14b the dissipation of the density functions is confined to a particular direction according to the GMTI sensor model.

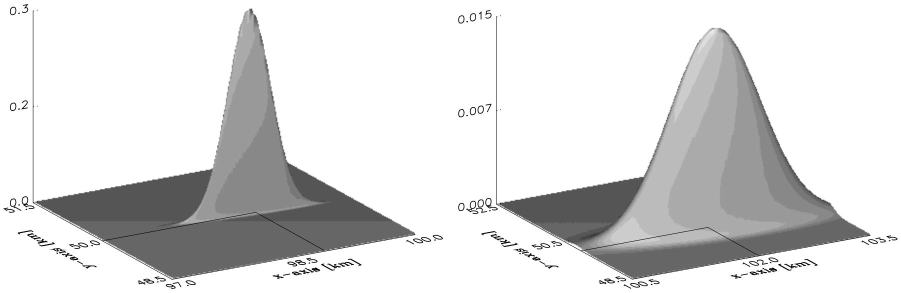

Fig. 2.15. Gain by fusing GMTI data from sensor 1 and 2: (a) during tracking. (b) target stop.

Gain by Sensor Data Fusion

Figures 2.13 – 2.15 show the probability densities of the target position in Cartesian ground coordinates after filtering. The prolated structure of the probability densities mirrors the predominant impact of cross-range errors. Their shape is rotated with respect to each other due to the different sensor-to-target geometries. This effect can be much more pronounced in other situations. We indicated the true target position. Figures 2.13a – 2.15a refer to a regular tracking situation (after 10 min, see Figure 2.6. Doppler-blindness occurred for sensor 2 during the previous revisits. The probability densities shown in Figure 2.13b – 2.15b have been calculated at a time when the target has stopped for 3 min. Evidently in Figures 2.13b, 2.14b the dissipation of the density functions is confined to a particular direction according to the GMTI sensor model.

Figure 2.15 shows the probability densities obtained by sensor data fusion. In both cases we observe a significant fusion gain. It is a consequence of the different orientation of the density functions and leads to improved state estimates. The result for the stopping targets is particularly remarkable. Though no sensor data are available from both sensors, the very fusion of the sensor output 'target under track is no longer detected' implies an improved target localization. This is a consequence of the different target/sensor geometries.

Key Publication

A detailed discussion of this approach has been published in:

- W. Koch and R. Klemm

  Ground Target Tracking with STAP Radar

  IEE Proceedings on Radar, Sonar and Navigation, Vol. 148, No. 3, p.173-185, June 2001 (Special Issue on: "Modeling and Simulation of Radar Systems, Ed.: S. Watts, invited paper).
  An extended version with results from various related conference papers of the author has been published as a handbook chapter in: W. Koch. Ground Target Tracking with STAP Radar: Selected Tracking Aspects. *Chapter 14 in: Klemm, R. (Ed.): Applications of Space-time Adaptive Processing. Institution of Electrical Engineers, IEE Press, 41 pages, London (2004).*

  Abstract
  The problem of tracking ground-moving targets with a moving radar (airborne, spaceborne) is addressed. Tracking of low Doppler targets within a strong clutter background is of special interest. The motion of the radar platform induces a spreading of the clutter Doppler spectrum so that low Doppler target echoes may be buried in the clutter band. Detection of such targets can be much alleviated by space-time adaptive processing (STAP) which implicitly compensates for the Doppler spread effect caused by the platform motion. Even if STAP is applied, low Doppler targets can be masked by the clutter notch. This physical phenomenon is frequently observed and results in a series of missing detections, which may seriously degrade the tracking performance. We propose a new sensor model adapted to STAP and discuss its benefits to tracking well-separated targets. By exploiting a priori information on the sensor specific clutter notch, the model in particular provides a more appropriate treatment of missing detections. In this context the Minimum Detectable Velocity (MDV) proves to be an important sensor parameter explicitly entering into ground-moving target indication (GMTI) tracking.

Key words: air-/spaceborne radar, STAP, GMTI radar, GMTI tracking, minimum detectable velocity (MDV), Bayesian target tracking, probabilistic data association (PDA)

2.3 Main-lobe Jamming

The degrees of freedom available in applications with airborne phased-array radar enable suppression of so called main-lobe jammers that try to blind the radar by transmitting specially designed radiation directly into the main beam of the radar, by using adaptive array signal processing techniques [128]. Following the spirit of the discussions in the previous sections, the current position of the resulting jammer notch as well as information on the distribution of the related monopulse measurements can be incorporated into a more sophisticated sensor performance model of air-borne phased-array radar. The proposed model does not only improve object tracking in the vicinity of a jammer notch in terms of a shorter extraction delay, improved track accuracy/continuity. It also has strong impact on strategies for adaptive sensor control.

2.3.1 Modeling the Jammer Notch

Tracking of an approaching missile under mail-lobe jamming conditions is among the most challenging data fusion tasks [129]. Advance sensor models can contribute to their efficient and robust solution. An example is the simulated situation in Figure 2.16, which shows the trajectories of a sensor (AESA: Active Electronically Scanned Array) on a moving platform (black), of an object to be tracked (red), and the jammer (magenta).

*

By using adaptive digital beamforming techniques, AESA radars of modern interceptor aircraft are able to electronically produce a sector of vanishing susceptibility in their receive beam pattern. Excepting this “blind spot”, also called jammer notch, the radar is operating more or less normally. A non-cooperative missile, however, is expected to approach the interceptor aircraft as long as possible in the shadow of the jammer notch. The dashed lines in Figure 2.16 characterize the spatial region of the blind spot depending on the current sensor-to-jammer geometry object.

The effect of the jammer is directly visible in the signal-to-noise-plus-jammer ration (SNJR) of the target, which is shown Figure 2.17 for the scenario discussed as a function of time. Only in the beginning can the missile be detected for a short time. Then it is masked for a long time by the radar’s blind spot, until it becomes visible again in close vicinity of the sensor, where the reflected signal is very strong (Burn Through). Sophisticated signal processing provides estimates of the missile direction by using adaptive monopulse techniques [128] as well as the corresponding estimation error covariance matrix

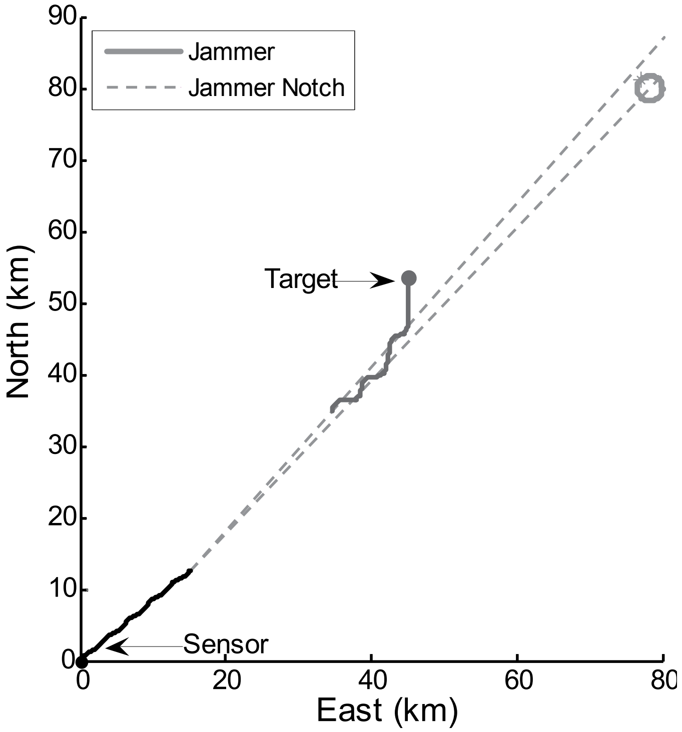

Fig. 2.16. Moving aircraft under mail-lobe jamming conditions: approaching missile near the shadow of the jammer notch

$\mathbf{R}(\mathbf{b}_k, \mathbf{j}_k)$ as an additional sensor output. $\mathbf{R}(\mathbf{b}_k, \mathbf{j}_k)$ depends on the current beam direction $\mathbf{b}_k$ of the AESA radar and the jammer direction $\mathbf{j}_k$ and describes in particular the mutual correlation of the estimated direction cosines in the vicinity of the jammer notch. It thus provides valuable context information on the sensor performance.

The sensor model is based on an expression for the signal-to-noise+jammer ratio (SNJR) after completing the signal processing chain. The following simple formula mirrors all relevant phenomena observed:

$$\mathrm{SNJR}(\mathbf{d}_k, r_k; \mathbf{b}_k, \mathbf{j}_k) = \mathrm{SNR}_0 \left(\tfrac{r_k}{r_0}\right)^{-4} D(\mathbf{d}_k) \times \\ \mathrm{e}^{-\log 2 |\mathbf{d}_k - \mathbf{b}_k|^2 / b^2} \left(1 - \mathrm{e}^{-\log 2 |\mathbf{d}_k - \mathbf{j}_k|^2 / j^2}\right).$$

The vectors $\mathbf{b}_k$ and $\mathbf{j}_k$ denote the angular position of the current beam and the jammer, respectively (assumed to be known). b is a measure of the beam width, while j indicates the width of the jammer notch produced by adaptive nulling, and r_0 is the radar's instrumented range. $\mathbf{d}_k$ is the object's direction vector and r_k its range from the sensor. $D(\mathbf{d}_d)$ reflects the antenna's directivity pattern. In the case of Swerling I fluctuations of the objects' radar cross section

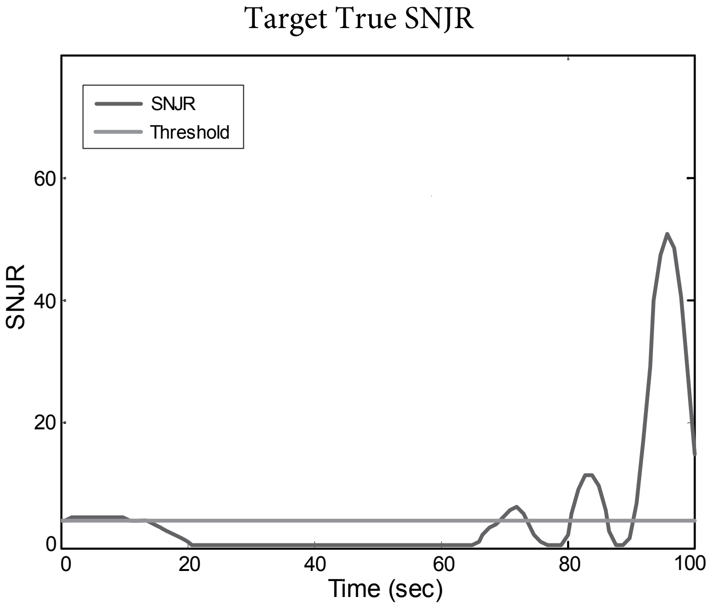

Fig. 2.17. Temporal variation of the signal-to-noise ratio under of an approaching missile under main-lobe jamming

and for a simple detection model, the detection probability is a function of $\mathbf{d}_k$, r_k, $\mathbf{b}_k$, and $\mathbf{j}_k$:

$$P_D(\mathbf{d}_k, r_k; \mathbf{b}_k, \mathbf{j}_k) = P_F^{\frac{1}{1+\mathrm{SNJR}(\mathbf{x}_k;\mathbf{b}_k,\mathbf{j}_k)}}. \tag{2.33}$$

P_D can be approximated by using Gaußians linearly depending on the object state. Essentially, we enter this expression of the detection probability into the likelihood function in Equation **??**, yielding a Gaußian sum type expression for it.

2.3.2 Tracking Filters Alternatives

According to the previous discussion, the signal-to-noise-plus-jammer is essential in the modeling of the detection probability and thus enters into the likelihood function ratio. After some approximations, the likelihood function can be represented by a Gaußian mixture, finally leading to a version of the Gaußian sum filter. Since the number of mixture components grows in each update step, adaptive approximation schemes must be applied. By using Monte-Carlo-simulations five competing approaches have been evaluated and compared with each other:

1. *Method 1 (Fixed EKF).* This tracking filter serves as a reference and uses no sophisticated sensor model. The impact of the jammer notch on P_D and the measurement error covariance matrix $\mathbf{R}$ are not taken into account.

2. *Method 2 (Variable EKF).* Here, only the monopulse error covariance $\mathbf{R}(\mathbf{b}_k, \mathbf{j}_k)$ is used as an improvement of the sensor model. The detection probability P_D is is assumed to be constant.
3. *Method 3 (Fixed Pseudo-bearing EKF).* This approach assumes a constant error covariance matrix $\mathbf{R}$, but uses the correct likelihood function, i.e. the jammer notch, in a second-order approximation.
4. *Method 4 (Variable Pseudo-bearing EKF).* In addition to the previous realization, here also the covariance matrix $\mathbf{R}(\mathbf{b}_k, \mathbf{j}_k)$ is part of the sensor model.
5. *Method 5 (Gaussian Sum Filter).* In this tracker the complete likelihood function and the monopulse covariance $\mathbf{R}(\mathbf{b}_k, \mathbf{j}_k)$ is used. The number of the mixture components involved to represent $p(\mathbf{x}_k|Z^k)$ is confined by three.

For the methods 3-5 the following is true: If the radar beam points to the vicinity of the blind spot and no detection occurs, a local search is performed. By this, probability mass is concentrated near the blind spot provided the target is actually there.

2.3.3 Selected Simulation Results

Figure 2.18 shows the mean track continuity averaged over 250 Monte-Carlo runs. The superiority of tracking methods that use context information on the spatial position of the blind spot is obvious. The use of the monopulse covariance matrix is necessary, but not sufficient for avoiding track loss. The methods 3, 4, and 5 can, using "negative" sensor evidence, bridge over the missing data in the jammer notch. In spite of the fact that method 5 is more computationally intensive than method 4, it shows deficiencies if compared with method 4. This is an indication for the fact that further performance improvements are possible by more advanced approximation methods.

Key Publication

A detailed discussion of this approach has been published in:

- W. Blanding, W. Koch , U. Nickel

  Adaptive Phased-Array Tracking in ECM Using Negative Information

  IEEE Transactions on Aerospace and Electronic Systems, vol. 45, nr. 1, p. 152-166, January 2009.

  Abstract

  Advances in characterizing the angle measurement covariance for phased array monopulse radar systems that use adaptive beamforming to null a jammer source allow for the use of improved sensor models in tracking algorithms. Using a detection probability likelihood function consisting of

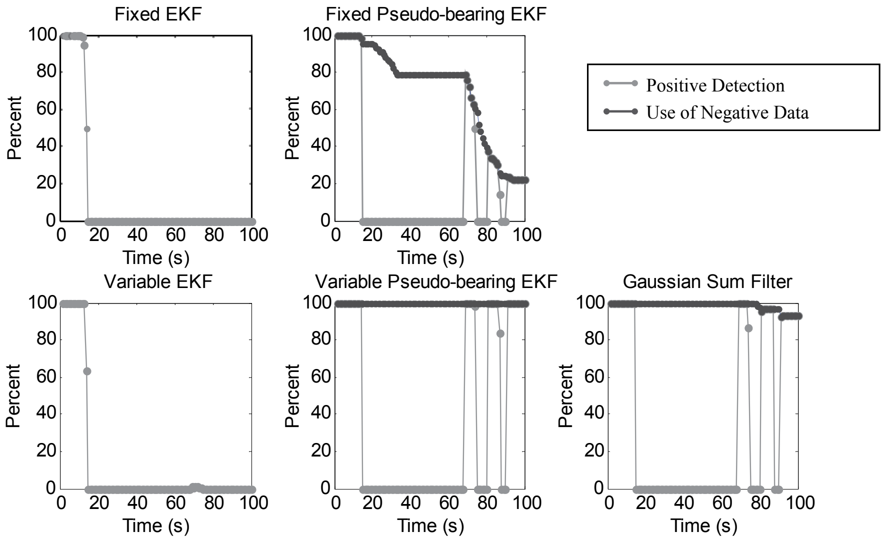

Fig. 2.18. Simulation results (250 runs) characterizing track continuity for different tracking filters.

a Gaussian sum that incorporates negative contact measurement information, four tracking systems are compared when used to track a maneuvering target passing into and through standoff jammer interference. Each tracker differs in how closely it replicates sensor performance in terms of accuracy of measurement covariance and the use of negative information. Only the tracker that uses both the negative contact information and corrected angle measurement covariance is able to consistently reacquire the target when it exits the jammer interference.

Keywords: Target tracking, adaptive beamforming, standoff jamming, Gaussian sum filter.

2.4 Negative Sensor Information

More advanced sensor models especially enable the exploitation of ‘negative’ sensor evidence. By this we mean the rigorous drawing of conclusions from expected but actually missing sensor measurements. These conclusions aim at an improvement of the position or velocity estimates for objects currently kept under track. Even a failed attempt to detect an object in the field of view of a sensor is to be considered as a useful sensor output, which can be processed by using appropriate sensor models, i.e. by background information on the sensors, with benefits for target tracking, sensor management, and sensor data fusion. The technical term chosen here for denoting such pieces of evidence, i.e. ‘negative’ information, seems to be accepted in the data fusion community (see, e.g. [132, 133]).

2.4.1 A Ubiquitous Notion

A very simple example illustrates that negative sensor information is an ubiquitous phenomenon, which often appears in disguise. The notion fits well into the Bayesian formalism. Assume a sensor producing at discrete time instants t_k mutually independent measurements $\mathbf{z}_k$ of a single object with Gaußian likelihood $\mathcal{N}(\mathbf{z}_k;\, \mathbf{H}\mathbf{x}_k,\, \mathbf{R})$. Absence of clutter is assumed ($\rho_F = 0$). The objects are detected with a constant detection probability $P_\mathrm{D} < 1$. We thus have classical Kalman filtering under the constraint that there exists not at each time a measurement. The likelihood function is thus given by Equation **??** and yields:

1. In the case of a positive sensor output ($m_k = 1$), $\mathbf{z}_k$ is processed by Kalman filtering leading to $p(\mathbf{x}_k|\mathcal{Z}^k) = \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$ with $\mathbf{x}_{k|k}$ and $\mathbf{P}_{k|k}$ given by:

$$\mathbf{P}_{k|k} = (\mathbf{P}_{k|k-1}^{-1} + \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{H})^{-1} \tag{2.34}$$

$$\mathbf{x}_{k|k} = \mathbf{P}_{k|k}(\mathbf{P}_{k|k-1}^{-1} \mathbf{x}_{k|k-1} + \mathbf{H}^\top \mathbf{R}^{-1} \mathbf{z}_k). \tag{2.35}$$

2. For a negative sensor output ($m_k = 0$), the likelihood function is a constant $1 - P_\mathrm{D}$. By filtering the prediction density is not modified: $\mathbf{x}_{k|k} = \mathbf{x}_{k|k-1}$, $\mathbf{P}_{k|k} = \mathbf{P}_{k|k-1}$. According to 2.34 and 2.35 this result could formally be interpreted as the processing of a fictitious measurement with an infinite measurement error covariance $\mathbf{R}$, since $\mathbf{R}^{-1} = 0$.

2.4.2 Lessons Learned from Examples

The Bayes formalism and the likelihood function thus precisely indicate, in which way a negative sensor output, i.e. a missing detection has to be processed. This observation notion can be generalized and leads to the following conclusions:

1. Missing but expected (i.e. negative) sensor data can convey information on the current target position or a more abstract function of the kinematic object state. This type of negative evidence can be included in data fusion within the rigorous Bayesian structure. There is no need for recourse to ad hoc or empirical schemes.
2. The prerequisite for processing negative evidence is a refined sensor model, which provides additional background information for explaining its data. As a consequence, negative evidence often appears as an artificial sensor measurement, characterized by a corresponding measurement matrix and a measurement error covariance.

3. The particular form of the fictitious measurement equation involved is determined by the underlying model of the sensor performance, while the fictitious measurement error covariance is characterized by sensor parameters such as sensor resolution, radar beam width, or minimum detectable velocity.
4. Negative evidence implies well-defined probability densities of the object states that prove to be Gaussian mixtures with potentially negative coefficients summing up to one. Intuitively speaking, these components reflect that the targets keep a certain distance from each other, from the last beam position, or the clutter/jammer notch.
5. If the fictitious measurement depends on the underlying sensor-to-target geometry, we can even introduce the fusion of negative evidence.

Key Publication

A detailed discussion of this approach has been published in:

- W. Koch

  On exploiting 'negative' sensor evidence for target tracking and sensor data fusion

  International Journal on Information Fusion, Volume 8, Issue 1, p.28-39, Elsevier, January 2007 (Special Issue: "Best Papers of FUSION 2004", Eds: P. Svensson, J. Schubert, invited paper).

  Abstract

  In various applications of target tracking and sensor data fusion all available information related to the sensor systems used and the underlying scenario should be exploited for improving the tracking/fusion results. Besides the individual sensor measurements themselves, this especially includes the use of more refined models for describing the sensor performance. By incorporating this type of background information into the processing chain, it is possible to exploit 'negative' sensor evidence. The notion of 'negative' sensor evidence covers the conclusions to be drawn from expected but actually missing sensor measurements for improving the position or velocity estimates of targets under track. Even a failed attempt to detect a target is a useful sensor output, which can be exploited by appropriate sensor models providing background information. The basic idea is illustrated by selected examples taken from more advanced tracking and sensor data fusion applications such as group target tracking, tracking with agile beam radar, ground-moving target tracking, or tracking under jamming conditions.

  Keywords: Negative information/evidence, target tracking, sensor resolution, local search, adaptive beam positioning, GMTI sensor fusion

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